Math Explorations

Part 1 Workbook

2013 Edition

Hiroko Warshauer

Terry McCabe

Max Warshauer

Published by

Stipes Publishing L.L.C.
204 W. University Ave.
Champaign, Illinois 61820

Copyright © 2013
TEXAS Mathworks

Publisher: Stipes

Authors: Hiroko Warshauer, Terry McCabe, and Max Warshauer

Project Coordinator: Hiroko Warshauer

Project Designer: Namakshi P. Kant

Contributing Teachers: Alexandra Eusebi, Krystal Gaeta, Diana Garza, Denise Girardeau, Sandra Guerra, Melinda Kniseley, Melissa Lara, Terry Luis, Barbara Marques, Pat Padron, Rolinda Park, Eduardo Reyna, Patricia Serviere, Elizabeth Tate, Edna Tolento, Lorenz Villa, Amanda Voigt, Amy Warshauer, Elizabeth Weed.

Editorial Assistance, Problem Contributions, and Technical Support:

Sam Baethge, Ashley Beach, Molly Bending, Tina Briley, Cheyenne Crooks, Rachel Carroll, Edward Early, Michelle Frei, Vanessa Garcia, Kristen Gould, Marie Graham, Michael Kellerman, Rhonda Martinez, Cody Patterson, Karen Vazquez, Alexander White, Sally Williams, Luis Sosa.

Sponsors: RGK Foundation, Kodosky Foundation, Meadows Foundation, and Intel Foundation

Copyright © 2013 Texas State University – Mathworks. All rights reserved.

ISBN: 978-1-938858-03-1.

No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of Texas Mathworks. Printed in the United States of America.

For information on obtaining permission for use of material in this work, please submit written request to Texas Mathworks, 601 University Drive, San Marcos, TX 78666. Fax your request to 512-245-1469, or email to mathworks@txstate.edu.

MATH EXPLORATIONS PART 1 WORKBOOK

table of contents

CH. 1 EXPLORING INTEGERS 1

Section 1.1 Building Number Lines 1
Section 1.2 Less Than and Greater Than 5
Section 1.3 Applications of the Number Line 9
Section 1.4 Distance Between Points 15
Section 1.5 Addition of Integers 19
Section 1.6 Subtraction of Integers 25
Section 1.7 Variables and Expressions 31
Section 1.8 Graphing on the Coordinate System 35
Chapter 1 Spiral Review 41

CH. 2 MULTIPLYING AND DIVIDING 45

Section 2.1 Skip Counting with Integers 45
Section 2.2 Area Model for Multiplication 53
Section 2.3 Linear Model for Division 59
Section 2.4 The Division Algorithm 65
Section 2.5 Long Division 69
Chapter 2 Spiral Review 73

CH. 3 FACTORS AND MULTIPLES 77

Section 3.1 Factors, Multiples, Primes, and Composites 77
Section 3.2 Exponents and Order of Operations 89
Section 3.3 Prime Factorization 97
Section 3.4 Common Factors and the GCF 103
Section 3.5 Common Multiples and the LCM 111
Chapter 3 Spiral Review 119

Copyright © 2013 Texas State Mathworks

MATH EXPLORATIONS PART 1 WORKBOOK

table of contents

CH. 4 FRACTIONS 125

Section 4.1 Models for Fractions 125
Section 4.2 Comparing and Ordering Fractions 139
Section 4.3 Unit, Mixed, Proper and Improper Fractions 145
Section 4.4 Addition and Subtraction of Fractions 153
Section 4.5 Common Denominators and Mixed Fractions 165
Chapter 4 Spiral Review 173

CH. 5 DECIMAL AND PERCENT REPRESENTATIONS 177

Section 5.1 Constructing Decimals 177
Section 5.2 Operating with Decimals 187
Section 5.3 Numbers as Decimals and Fractions 197
Section 5.4 Fractions, Decimals, and Percents 203
Chapter 5 Spiral Review 211

CH. 6 EQUATIONS, INEQUALITIES, AND FUNCTIONS 215

Section 6.1 Patterns and Sequences 215
Section 6.2 Equations 223
Section 6.3 Equations and Inequalities on Number Lines 233
Section 6.4 Functions 237
Chapter 6 Spiral Review 249

CH. 7 RATES, RATIOS, AND PROPORTIONS 253

Section 7.1 Multiplying Fractions 253
Section 7.2 Division of Fractions 259
Section 7.3 Rates and Ratios 265
Section 7.4 Proportions 271
Chapter 7 Spiral Review 279

Copyright © 2013 Texas State Mathworks

MATH EXPLORATIONS PART 1 WORKBOOK

table of contents

CH. 8 MEASUREMENT 283

Section 8.1 Length 283
Section 8.2 Capacity and Volume 289
Section 8.3 Weight and Mass 295
Section 8.4 Time and Temperature 301
Chapter 8 Spiral Review 305

CH. 9 GEOMETRY 309

Section 9.1 Measuring Angles 309
Section 9.2 Triangles 323
Section 9.3 Quadrilaterals and Other Polygons 333
Section 9.4 Perimeter and Area 343
Section 9.5 Circles 355
Section 9.6 Three-Dimensional Shapes 365
Chapter 9 Spiral Review 377

CH. 10 DATA ANALYSIS 381

Section 10.1 Measures of Central Tendency 381
Section 10.2 Graphing Data 397
Section 10.3 Probability 405
Section 10.4 Rule of Product and Rule of Sum 415
Chapter 10 Spiral Review 423

CH. 11 MATH OF FINANCE 427

Section 11.1 Types of Charge Cards 427
Section 11.2 Credit Reports 431
Section 11.3 Going to College 433

Copyright © 2013 Texas State Mathworks

EXPLORING INTEGERS 1

Name:___________________________ **Date:**_______ **Period:**______

SECTION 1.1 BUILDING NUMBER LINES

VOCABULARY

DEFINITION	EXAMPLE
Number Line Model:	
Counting Numbers/Natural Numbers/Positive Integers:	
Whole Numbers:	
Integers:	
Origin:	

Big Idea: How do we categorize numbers? How do we construct number lines?

EXPLORATION: CONSTRUCTING A NUMBER LINE

1. Draw a straight line.
2. Pick a point on the line and call this point the origin. Label the origin with the number 0.
3. Locate the numbers 1, 2, 3, … 10, and -1, -2, -3, …, -10.

4. Where would 20, 30, 50 be located? 100? 1,000?
5. Find the negative numbers corresponding to the numbers in question 4.

PROBLEMS:

1. At the mall, the card shop is at the origin. Label it as point 0. The Ice cream Shop is located 5 units to the right of the origin. The pet shop is 3 units to the left of the Ice cream Shop. There is a sunglasses shop 4 units to the right of the jeans warehouse, and the jeans warehouse is 8 units to the left of the origin. Draw a number line representing the mall. Label each of the locations on the number line. **Watch your spacing.**

2. Write an integer for each situation. Find the point on the number line that corresponds to the integer. (Create a number line from -15 to 15, counting by fives and leaving a mark for each integer.)

a. a deposit of $14
b. 6° below zero
c. descend 11 yards
d. a growth of 3"

3. Chris visits Edmonton, Canada where it is -7° C. Carmen visits Winnipeg, Canada where it is 9° C. Which temperature is closer to the freezing point? Draw a thermometer to prove your answer. Remember, when we measure temperature in degrees Celsius, 0°C is the freezing point of water.

4. For each of the following pairs of integers, decide which integer is further to the right on the number line. (Circle your answer) Show why on the number line.

   a. 7 and 9
   b. 132 and 28
   c. 9 and -14
   d. -5 and -7
   e. 4 and -22
   f. -9 and -10
   g. -255 and -217
   h. -11 and -15

5. Make a list of words that indicate a positive or negative integer.

Negative Key Words	**Positive Key Words**
Example: withdraw	Example: deposit

6. Name each number as an integer, whole number, counting number, or none of the above. Use all the names that apply.

$$7,\ \frac{1}{7},\ -7,\ 0,\ 101,\ -1,\ 1$$

SUMMARY (What I learned in this section)

EXPLORING INTEGERS 1

Name:__________________________ Date:_______ Period:______

SECTION 1.2 LESS THAN AND GREATER THAN

VOCABULARY

DEFINITION	EXAMPLE
Less Than:	
Greater Than:	
Inequality:	
Variable:	

Big Idea: How do we compare and order integers?

EXAMPLE 1:

For each pair of integers below, imagine where they would be located on a number line and determine which one is greater and which one is less. Express your answer as an inequality of the form $x < y$, where x and y are the given integers.

a. 3 ☐ 7

b. $^-2$ ☐ 9

c. $^-1$ ☐ $^-5$

d. 4 ☐ $^-4$

EXAMPLE 2:

Create a number line to put the following integers in order from least to greatest:

$2,\ ^{-}2,\ 7,\ ^{-}1,\ ^{-}4,\ ^{-}5,\ 4,\ 6,\ 3$

PROBLEMS:

1. Put the following integers in order from greatest to least: -3, -1, 0, -11, 2, -6, 5

2. Rewrite each of the following as a statement using $<$ or $>$. Compare your statements to the relative locations of the two numbers on a number line. Example: -3 is less than 8. $-3 < 8$

   a. 10 is greater than 5 ____________________
   b. -2 is greater than -8 ____________________
   c. 4 is greater than 0 ____________________
   d. -6 is less than 5 ____________________
   e. 0 is greater than -210 ____________________

3. Compare the numbers below and decide which symbol, $<$ or $>$, to use between the numbers. In each show the relationship of these numbers on a number line.

   a. 7 ☐ 4
      4 ☐ 7

   b. 0 ☐ 9
      9 ☐ 0

   c. -8 ☐ 0
      0 ☐ -8

   d. -7 ☐ -6
      -6 ☐ -7

4. What are the possible values for an integer that is greater than -1 and less than 5. Show these values on a number line.

5. In St. Louis, Missouri, the morning temperature is -2° C. In the evening, the temperature reads -7° C. Did the temperature rise or fall? Draw a thermometer to show how much the temperature rose or fell.

SUMMARY (What I learned in this section)

Name:____________________________ Date:_______ Period:______

SECTION 1.3 APPLICATIONS OF THE NUMBER LINE

VOCABULARY

DEFINITION	EXAMPLE
Timeline:	

Big Idea: How can number lines be applied in a real-world use?

EXPLORATION 1: CONSTRUCTING A TIMELINE

In this activity, we will construct a special kind of number line called a **timeline**. Let's begin by building a timeline that goes back 100 years and forward 50 years. The first step is to draw a number line and label the origin with 0. The origin corresponds to the present year. Write the year above the line and the length of time from this year (our zero year) below the same mark. We want to label our timeline so that years that have already passed are labeled with negative numbers and years in the future are labeled with positive numbers.

1. What scale should we use?

2. How many marks did you decide to use on your timeline?

3. Plot the special dates that you previously gathered from home.

4. Make a timeline that goes back 3,000 years and forward in time one thousand years. How long is the total span of this timeline?

EXPLORATION 2: DISTANCES ON MAIN STREET

1. Look at the number line below, and label it Main Street.

   a. Plot the following locations on Main Street: the post office at address 0; the laboratory at address 6; the zoo at address 9; the candy shop at address $^{-}4$; and the space observatory at address $^{-}7$.

   b. What is the distance of each location mentioned above from the post office?

2. Using your number line for Main Street from part 1, find the following distances:

   a. The distance between the laboratory and the zoo. __________
   b. The distance between the space observatory and the candy shop. __________
   c. The distance between the zoo and the candy shop. __________
   d. The distance between the space observatory and the laboratory. ___________
   e. The distance between the space observatory and the zoo. ___________
   f. The distance between the laboratory and the candy shop. ___________

Discuss how you determine the distance between two locations on the number line

____

____

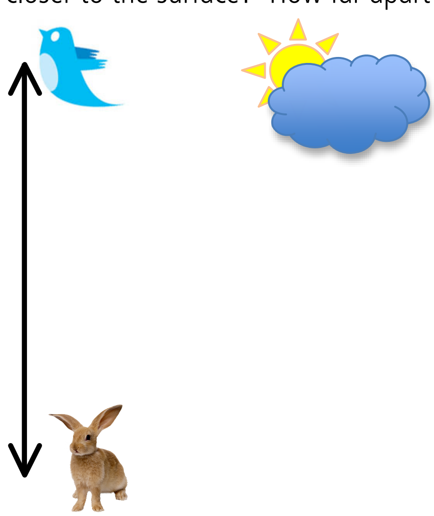
____

PROBLEMS:

1. The temperature in Eugene, Oregon, is 32° F. The temperature in Soldotna, Alaska is 13° F. Which city has the colder temperature? How many degrees colder is that city?

2. Diver Stuart reached a depth of 175 feet below the surface of the water, Diver Eric reached a depth of 212 feet. Which diver is farther from the surface? What is the distance between the two divers? Draw a picture to explain your answer.

3. A bird is flying 14 feet above the ground. A rabbit is burrowed 14 feet underground. Which is closer to the surface? How far apart are the animals? Explain your answer.



4. Which of the following integers is farthest from 17: 8, -1, 22, or -6? Draw a number line to prove your answer.

5. Draw a number line and find the distances between the following pairs of numbers on a number line.

   a. 9 and 5 b. -7 and -12 c. 7 and -3 d. -2 and 5

6. Draw a number line, and mark the integers -10 to 10.
    a. Name two integers that are two units from zero.
    b. Name two integers that are 3 units away from -6.

7. Name two integers that are 48 units away from 1,812.

SUMMARY (What I learned in this section)

EXPLORING INTEGERS 1

Name:___________________________ **Date:**_______ **Period:**______

SECTION 1.4 DISTANCE BETWEEN POINTS

VOCABULARY

DEFINITION	EXAMPLE
Absolute value:	
Magnitude:	

Big Idea: What are absolute values of numbers? How do we find distance between points on a number line?

PROBLEM 1:

Locate the points 5, -1, 4, -8, -5, -4, 1, and 8 on the number line below.

1. Write the pairs of numbers that are the same distance from the origin, 0.

______________ ______________ ______________ ______________

2. Find the absolute values of each number and determine which are equal. Write these equal pairs below using the absolute value notation. For example, $|-7| = |7|$.

PROBLEM 2:

Evaluate each of the following.

|-3| = |3| =   -|3| =   -|-3| =

PROBLEM 3:

For each pair of numbers below, place the correct symbol < , > , or = between them.
Be able to explain your choice of answer.

a. |-4| |3| b. |-9| |-8| c. -9   -8

d. 0 |-4| e. 0   -4   f. –(-2) |-2|

EXPLORATION: FINDING DISTANCE BETWEEN POINTS

1. Pick a point on the number line below and call this point the origin. Label the origin with the number 0.
2. Locate the numbers 1, 2, 3, … 10, and -1, -2, -3, …, -10.

3. Use the number line to find the distance between each pair of numbers.

a. 0 and 5   d. 1 and 5   g. 3 and 9

b. -0 and -5   e. 1 and -5   h. -3 and -9

c. 5 and -5   f. -1 and 5   i. -3 and 9

4. What relationship do you see between distance between points and the absolute value?

SUMMARY (What I learned in this section)

Name:___________________________ **Date:**_______ **Period:**______

SECTION 1.5 ADDITION OF INTEGERS

VOCABULARY

DEFINITION	EXAMPLE
Additive Identity:	
Additive Inverse:	
Commutative Property of Addition:	
Associative Property of Addition:	
Double Opposite Theorem:	

Big Idea: How do we model addition on the number line?

CLASS EXPLORATION: DRIVING ON THE NUMBER LINE WITH ADDITION

We can visualize adding two numbers using a car driving on the number line and the **Four-Step Car Model**. The final location gives the sum. Use a number line from -15 to 15 as your highway. You will also need a model car or an object that can represent this model car (for example, an eraser).

Four-Step Car Model

Step 1: Place your car at the origin, 0, on the number line.

Step 2: If the first of the two numbers is positive, the car faces right or the positive direction. If the first of the two numbers is negative, the car faces left or the negative direction. Drive to the location given by this first number and park the car.

Step 3: Examine the second of the two numbers. If this number is positive, point the car to the right, the positive direction. If the second number is negative, point the car to the left, the negative direction.

Step 4: Move the car forward (because you are adding), in the direction that the car is facing, the distance equal to the absolute value of the second number.

EXAMPLE 1

Use the Four-Step Car Model to find the sum $3 + 4$ on the number line and describe your four steps carefully**.** Show your process on the number line.

Step 1:

Step 2:

Step 3:

Step 4:

EXAMPLE 2

Use the Four-Step Car Model to find the sum -3 + 4. Describe your four steps and show the process on the number line.

Step 1:

Step 2:

Step 3:

Step 4:

EXAMPLE 3

Use the Four-Step Car Model to find the sum -3 + (-4), or simply -3 + -4 using the number line and explain your reasoning for how you found the sum.

Step 1:

Step 2:

Step 3:

Step 4:

PROBLEMS

Model the sums using your car on the number line.

1. 4 + 7

2. -4 + -7

3. 4 + -7

4. -4 + 7

PROBLEM

Identify the **property of addition** that is illustrated in the equalities below.

1. -6 + 6 = 0 ______________________
2. -9 + 0 = -9 ______________________
3. 8 + 3 = 3 + 8 ______________________
4. 2 + (8 + 6) = (2 + 8) + 6 ______________________

SUMMARY (What I learned in this section)

EXPLORING INTEGERS 1

Name:____________________________ Date:________ Period:_______

SECTION 1.6 SUBTRACTION OF INTEGERS

Big Idea: How do we model subtraction on the number line?

CLASS EXPLORATION: DRIVING ON THE NUMBER LINE WITH SUBTRACTION

We can visualize adding two numbers using a car driving on the number line by using a Four-Step Car Model for subtraction. The final location gives the difference. Use a number line from -15 to 15 as your highway. You will also need a model car or an object that can represent this model car (for example, an eraser).

Four-Step Car Model for Subtraction

Step 1: Place your car at the origin, 0, on the number line.

Step 2: Look at the sign on the first number. If the number is positive, point the car to the right. If the number is negative, point the car to the left.

Step 3: Next, examine the sign on the second of the two numbers. If the number is positive, point the car to the right. If the number is negative, point the car to the left.

Step 4: Move the car **backward** (because you are subtracting), the distance equal to the absolute value of the second number.

EXAMPLE 1

Use the Four-Step Car Model to compute the difference 5 – 2 on the number line and describe your four steps carefully.

Step 1:

Step 2:

Step 3:

Step 4

EXAMPLE 2

Use the Four-Step Car Model to compute the difference 2 – 5. Use the number line to show how you solved the problem.

Step 1:

Step 2:

Step 3:

Step 4

EXAMPLE 3

Use the Four-Step Car Model to compute the difference -7 – 3. Use the number line to show how you solved the problem.

Step 1:

Step 2:

Step 3:

Step 4

EXAMPLE 4

Use the Four-Step Car Model to compute the difference 2 – (-5). Use the number line to show how you solved the problem.

Step 1:

Step 2:

Step 3:

Step 4

PROBLEM

In each case, model the differences using the Four-Step Car Model on the number line. Be able to explain the process for finding the difference in each case.

1. 4 – 7

2. -4 – (-7)

3. 4 – (-7)

4. -4 – 7

SUMMARY (What I learned in this section)

Name:___________________________ **Date:**_______ **Period:**______

SECTION 1.7 VARIABLES AND EXPRESSIONS

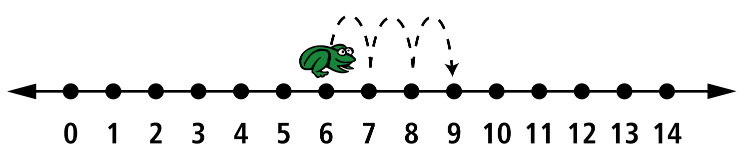
VOCABULARY

DEFINITION	EXAMPLE
Variable:	
Expressions:	

Big Idea: What are variables and expressions and how are they used in mathematics?

EXAMPLE 1:

Translate "three more than six" into a mathematical expression.



EXAMPLE 2:

Translate "two less than five" into a mathematical expression.

EXAMPLE 3:

Translate "five less than two" into a mathematical expression.

What is the difference between the expressions in Example 2 and Example 3?

EXAMPLE 4:

Translate "eight more than *x*" into a mathematical expression. Illustrate this on a number line with an arbitrary point as coordinate *x*.

What does *x* represent? ______________________________

PROBLEMS:

1. Translate each of the following into a mathematical expression.
   a. Three more than six __________

      Six more than three __________

      Three less than six __________

      Six less than three __________

   b. Two more than twenty __________

      Twenty more than two __________

      Two less than twenty __________

      Twenty less than two __________

2. Write the following mathematically.

   a. 12 decreased by 4 __________

   b. 8 less than 2 __________

   c. 5 years younger than an 18-year old __________

d. 13 increased by 18 ______________

e. *y* more than 62 _____________

f. *x* less than *w* _____________

3. Translate the following expressions or inequalities into word phrases or sentences.

   a. 12 – 3 ____

   b. -5 + 7 ____

   c. $r < 16$ ____

   d. $p > q$ ____

4. Translate each of the following into a mathematical expression or inequality:

   a. five less than thirty ____

   b. five is less than thirty ____

   c. *a* more than *d* ____

   d. *a* is greater than *d* ____

   e. *a* less than *c* ____

5. Charlie eats a dozen cookies and has 3 cookies left. Let C = the number of cookies Charlie had at the beginning. What integer does C represent? Explain how you arrived at your solution. You may use a number line to explain.

6. Explain the difference between the statements "is less than" and "less than". Write two expressions to support your answer.

7. Explain the difference between "six more than four" and "six is greater than four".

SUMMARY (What I learned in this section)

EXPLORING INTEGERS 1

Name:__________________________ Date:_______ Period:______

SECTION 1.8 GRAPHING ON THE COORDINATE SYSTEM

VOCABULARY

DEFINITION	EXAMPLE
Coordinate:	
Coordinate Plane or coordinate graph:	
Origin:	
Horizontal Axis or x-axis:	
Vertical Axis or y-axis:	
Axes:	
Quadrant:	
Ordered Pairs or coordinate pairs:	
x-coordinate:	
y-coordinate:	
Lattice Points:	

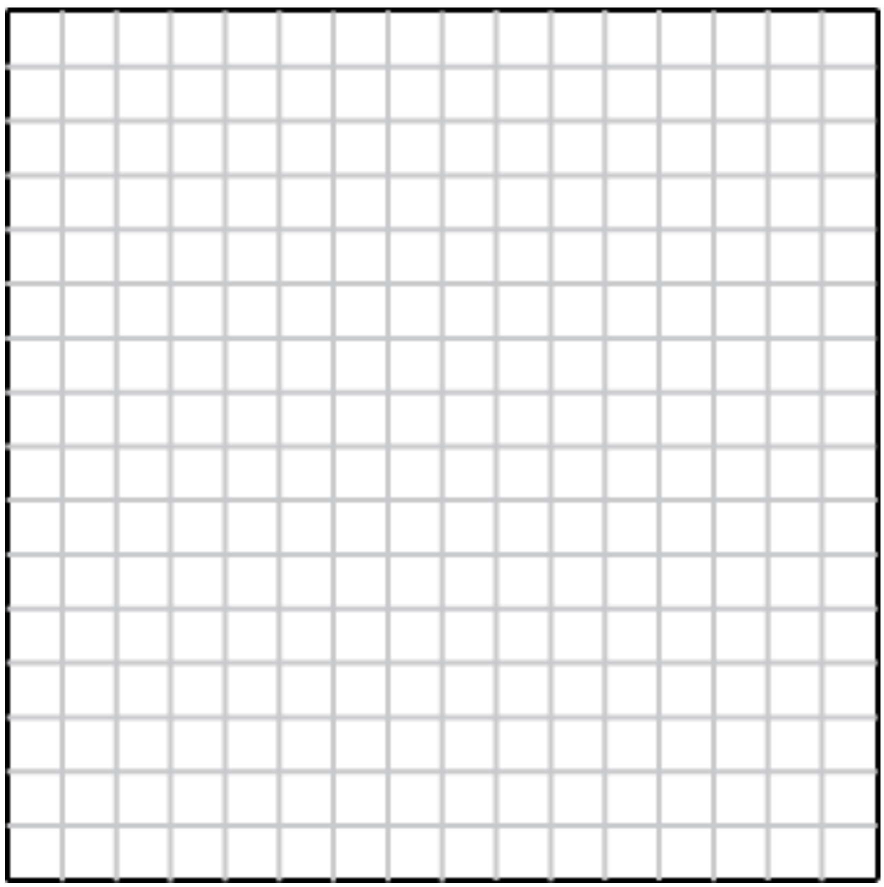
Big Idea: How do we graph points on a coordinate plane?

EXPLORATION 1:

On the graph paper below, draw a horizontal number line. Label the horizontal number line "**x-axis**". Label the **origin** in the middle and the integers on either side.

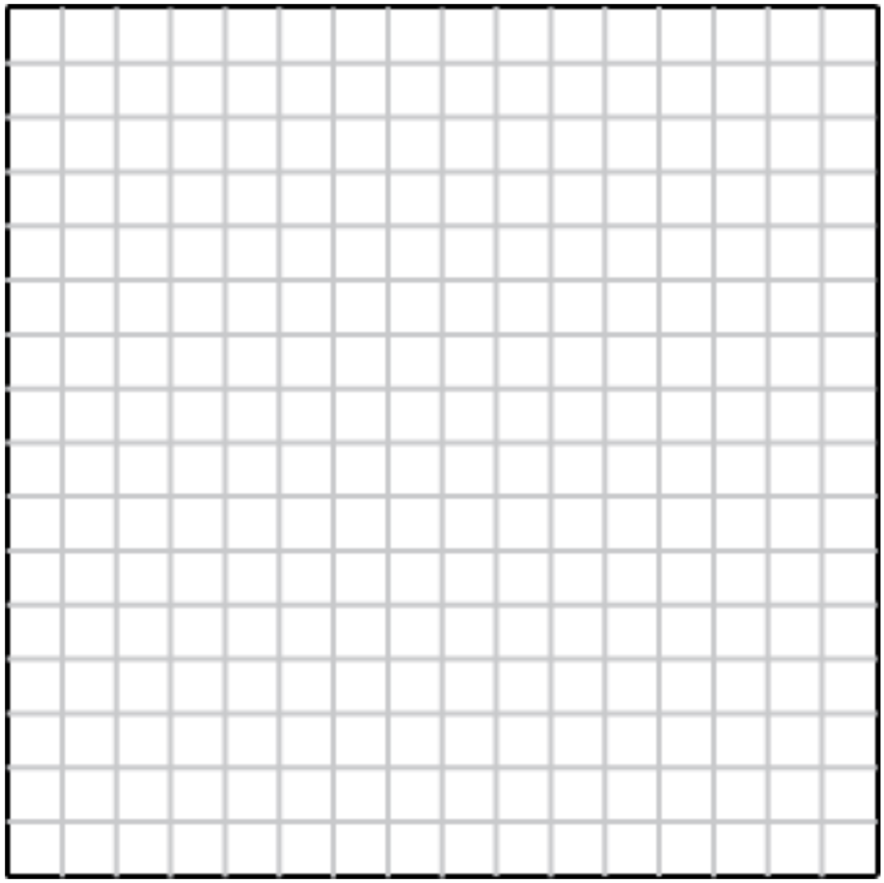
Draw a vertical line that intersects the horizontal number line at the origin, and label it **y-axis**. Mark the integers above and below zero.

Now you are ready to label the Quadrants. Begin in the upper right region and, using Roman Numerals, label "**Quadrant I**". Move counterclockwise to label **Quadrants II, III,** and **IV**.



EXPLORATION 2:

On the graph paper below, **construct and label** a coordinate plane as you did in Exploration 1.



a. Plot and label the following points on the coordinate plane. State which quadrants each point is in.

N (-5, 2) Quadrant ___

B (3, 5) Quadrant ___

M (-4, -3) Quadrant ___

S (4, -1) Quadrant ___

b. Name a point in each of the quadrants.

A point in Quadrant I : ____________

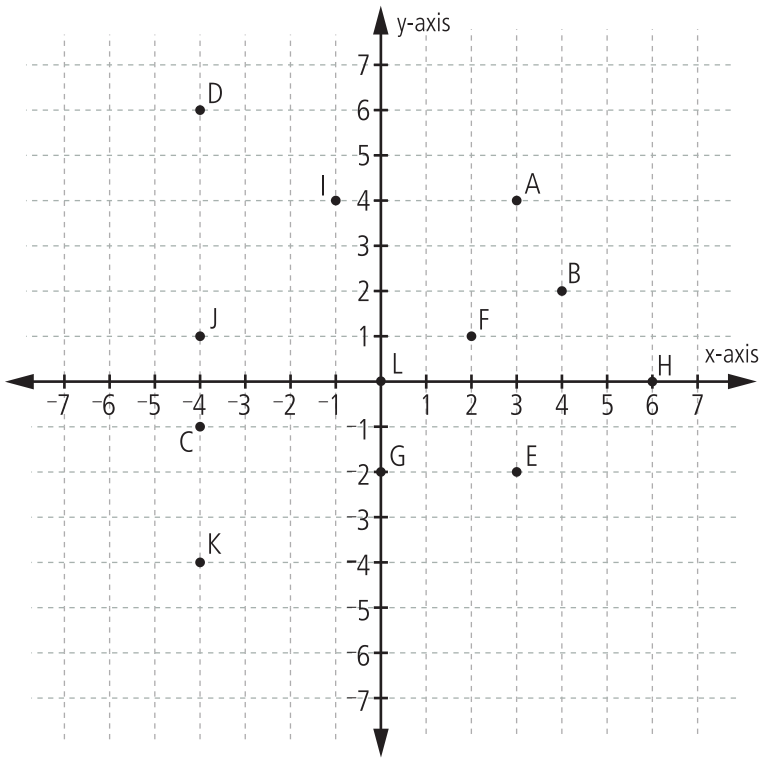
A point in Quadrant II : ____________

A point in Quadrant III : ____________

A point in Quadrant IV : ____________

PROBLEMS:

1. Write the coordinates for each point shown on the coordinate plane below.



A __________ G __________

B __________ H __________

C __________ I __________

D __________ J __________

E __________ K __________

F __________ L __________

2. Use the coordinate plane from Problem 1 to plot the following points.

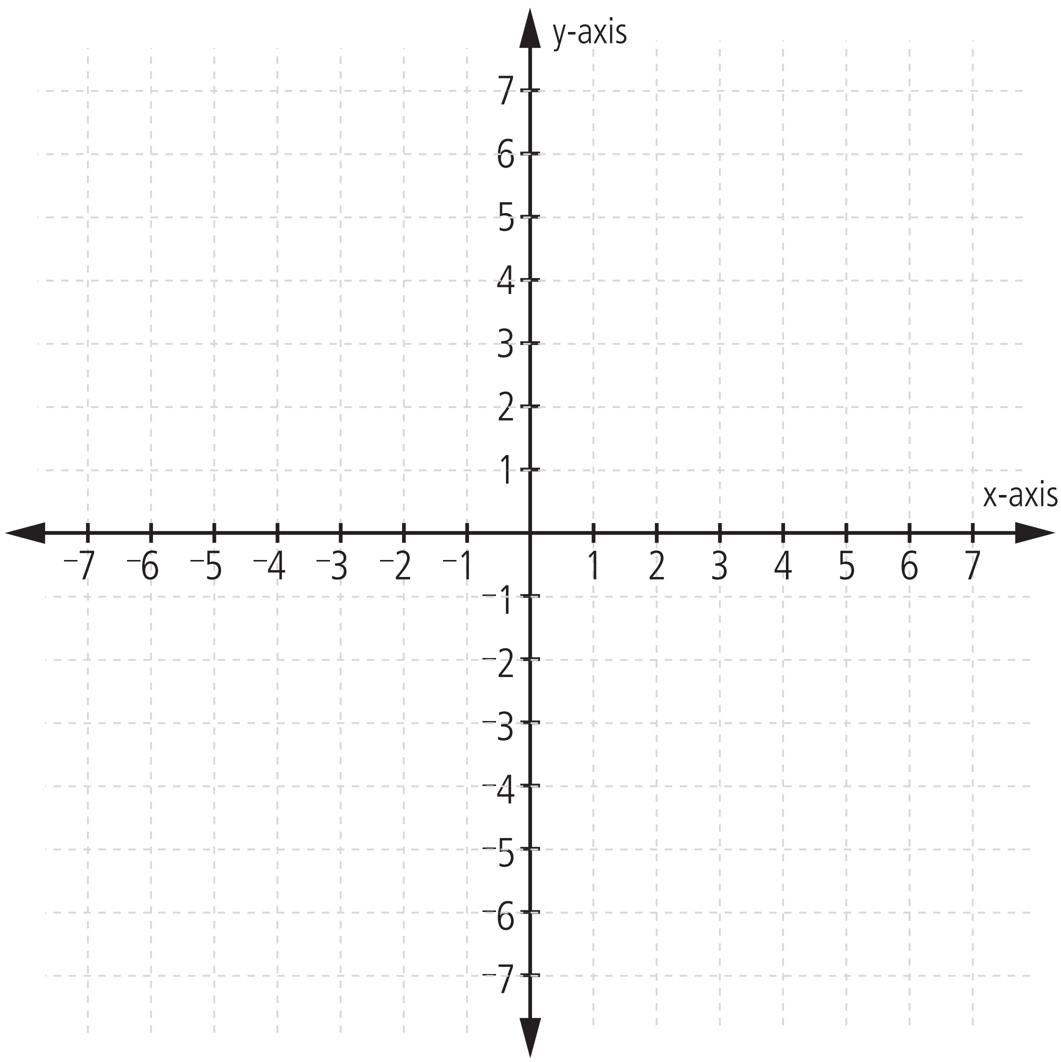
P (0, 3)	Q (-6,6)	R (2, 4)	S (-5, -3)
T (-3, 5)	U (3, 0)	W (1, 1)	Z (4, -5)

3. For each condition below make a list (or T-Chart) of 4 points. Plot these points on the coordinate plane below.

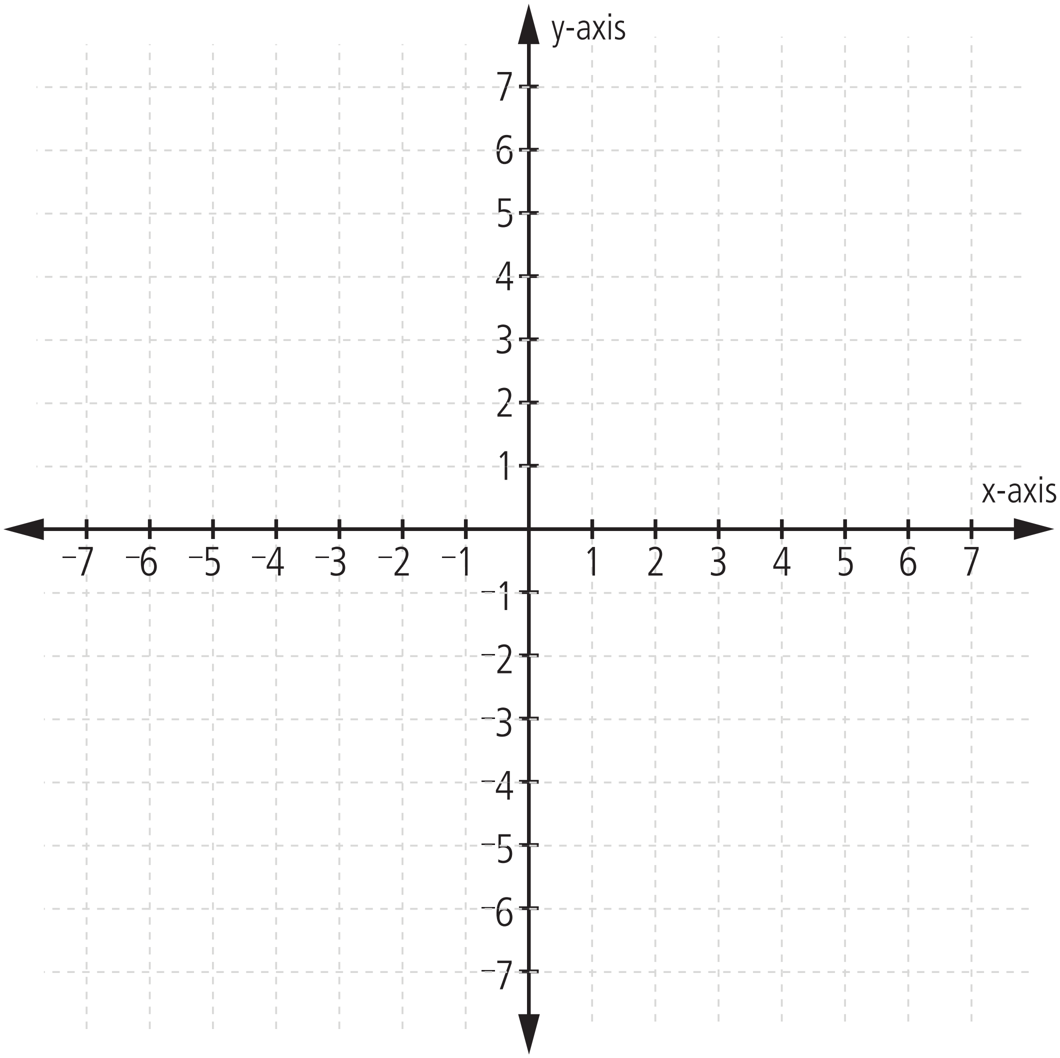
   a. Each point has an x-coordinate equal to zero and a positive y-coordinate.

   b. Each point has a y-coordinate that is double the x-coordinate.

   c. Each point has a y-coordinate that is equal to the x-coordinate.



4. On the coordinate plane below, draw a trapezoid that has a vertex in each quadrant. Label each vertex to name the trapezoid ABCD.



What ordered pairs represent the vertices of your trapezoid?

SUMMARY (What I learned in this section)

EXPLORING INTEGERS 1

Name:____________________________ Date:________ Period:_______

CHAPTER 1: SPIRAL REVIEW

1. Write < , > , or = into each blank.

a.	$5 + 2$ ______ $9 - 7$	b.	$14 + 6$ ______ $24 - 8$
c.	$12 + 20$ ______ $20 + 5 + 7$	d.	$118 \div 90$ ______ $88 + 21$
e.	$12 \cdot 6$ ______ $9 \cdot 10$	f.	$14 \cdot 3$ ______ $12 \cdot 4$
g.	$28 \cdot 2$ ______ $100 \div 2$	h.	$21 \cdot 7$ ______ $363 \div 3$

2. Myra is building a square chicken pen. Each side is 5 feet long. What is the perimeter of the pen?

____

3. Crystal walked 2 miles on Monday, 3 miles on Tuesday, and 4 miles on Wednesday. How many miles must she walk in the next four days to reach her goal of 20 miles?

____

4. Angel ran around the circumference of a round swimming pool five times. He ran a total of almost 100 feet. About how long is the distance around the pool?

____

5. Find the sum or difference. Use addition or subtraction to check your answers.

a. 1,453 + 64 =	b. 1,589 – 724 =	c. 594 + 169 =
d. 4,367 + 362 =	e. 3,671 – 354 =	f. 721 + 422 =
g. 1,270 + 955 =	h. 2,003 – 977 =	i. 655 + 681 =

6. Find the product or quotient. Use division or multiplication to check your answers.

a. $506 \cdot 39 =$	b. $945 \div 15 =$	c. $149 \cdot 80 =$
d. $612 \div 36 =$	e. $3{,}144 \cdot 28 =$	f. $728 \cdot 41 =$
g. $2{,}262 \div 78 =$	h. $36{,}975 \div 87 =$	i. $263 \cdot 19 =$

7. Use the number line below to find the distance between each of the following pairs of numbers.

a. 0 and -11 — The distance between 0 and -11 is _________.

b. -8 and 8 — The distance between -8 and 8 is __________.

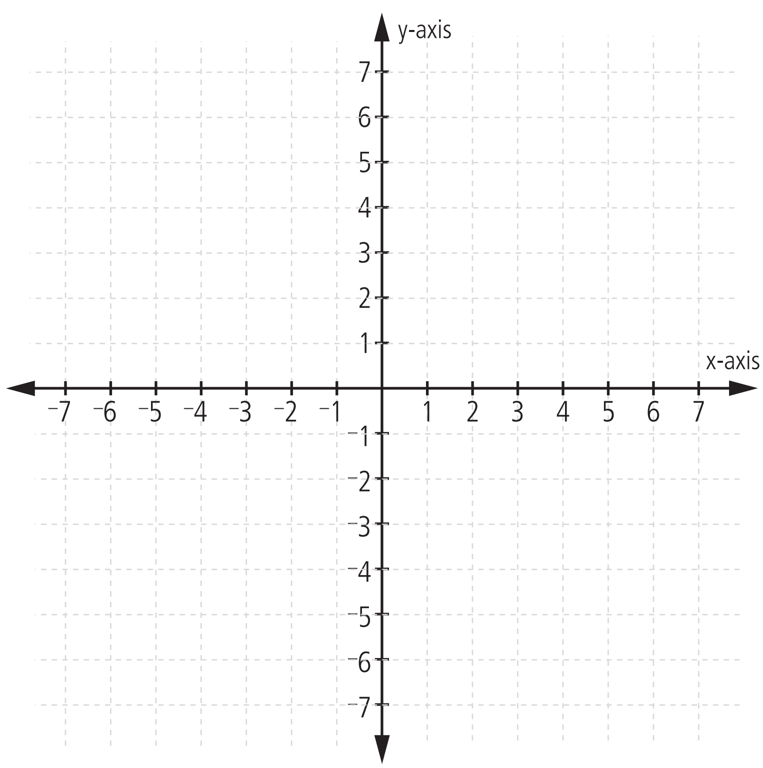
c. -3 and 32 — The distance between -3 and 32 is _________.

8. a. Which of the following numbers is farthest from 0: -5, 7, or -3?

b. Which of the following numbers is closest to 0: -28, 26 or -17?

9. Plot the following points on the coordinate plane below.

b. (6, 4) m. (-4, -3) n. (-7, 7) s. (6, -6)



10. Examine the ordered pairs listed below and identify the quadrant in which they would be found on the coordinate plane.

a. (46, -7) Quadrant ______

b. (-10, -8) Quadrant ______

c. (64, 90) Quadrant ______

d. (-11, 27) Quadrant ______

MULTIPLYING AND DIVIDING 2

Name:___________________________ **Date:**_______ **Period:**______

SECTION 2.1 SKIP COUNTING WITH INTEGERS

VOCABULARY

DEFINITION	EXAMPLE
Product:	
Factor:	

Big Idea: How do we model multiplication on a number line?

EXPLORATION 1: FROG JUMP MULTIPLICATION

Complete the Table 2.1a in which each jump is 4 units long.

Table 2.1a

Length of Jump (factor)	Number of Jumps (factor)	Frog's Location (product)
4	0	0
4	1	4
4	2	8
4	3	
4	4	
4	5	
4	6	
4	10	
4	20	
4	n	

Does this table look familiar? You might recognize these numbers from a multiplication table of 4's where the pattern is $4 \cdot 1 = 4$; $4 \cdot 2 = 8$; $4 \cdot 3 = 12$; $4 \cdot 4 = 16$. You can think of (4)(3) as (4 units per jump)(3 jumps) = 12 units.

Complete the Table 2.1b as you did in Table 2.1a, but this time use jumps of directed length 7.

Table 2.1b

Length of Jump	Number of Jumps	Frog's Location
7	0	0
7	1	7
7	2	
7	3	
7	4	
7	5	
7	6	
7	10	
7	20	
7	n	

Using the pattern demonstrated in this table, compute the product. Multiplication of 7 and 12, often written as 7×12, can also be written as $7 \cdot 12$, 7 *12, or (7)(12).

EXPLORATION 2:

In McAllen, TX, the temperature rises an average of 3° F per hour over a 12 hour period from 1 am to 1 pm. The temperature at 7 am is 72° F. Let *x* be the number of hours after 7am.

a. Complete the table below to show the relationship between time and temperature over the 12 hours.

Time	1 am	2 am	3 am	4 am	5 am	6 am	7 am	8 am	9 am	10 am	11 am	12 pm
# of hrs. *x*							0	1	2			
Temp.							72° F					

b. What was the temperature at 3 am?

c. What *x*-value corresponds to 11 pm?

d. When is the temperature 63° F?

e. Is it possible to use multiplication to help determine the temperature in parts b and c? If so, explain how.

PROBLEMS:

1. Write the expression five multiplied by fifteen using the different multiplication symbols.

2. Write the equation $5x = 30$ using words.

3. Compute the following products.

   a. (18) (9)   b. (20) (14)   c. (14) (y)   d. (115) (30)

Multiplication Model: In the product (m)(n), we think of the first factor m as the length and direction of each jump and we think of n as the number of jumps.

EXPLORATION 3:

Use jumps of directed length -4 to fill the skip counting in Table 2.1d. You may use the number line below to confirm the Frog's location.

Table 2.1d

Directed Length of Jump	Number of Jumps	Frog's Location
$^{-}4$	0	0
$^{-}4$	1	$^{-}4$
$^{-}4$	2	
$^{-}4$	3	
$^{-}4$	4	
$^{-}4$	5	
$^{-}4$	6	
$^{-}4$	10	
$^{-}4$	20	
$^{-}4$	n	

How can we make sense of the product (3) (-4)? This is the first example where the second factor is negative.

The first number, 3 or +3, gives the length of each jump, and the direction the frog is facing. Because the number is positive, the frog faces right.

The second factor gives the number of jumps. What do we mean by the number -4 as the number of jumps? If we think of the jumps taking place at equal time intervals, we can imagine the frog jumping along a line. You may use the number line below to confirm the Frog's location.

Table 2.1e

Directed Length of Jump	Number of Jumps	Frog's Location
3	$^{-}6$	
3	$^{-}5$	
3	$^{-}4$	
3	$^{-}3$	
3	$^{-}2$	
3	$^{-}1$	
3	0	0
3	1	3
3	2	6
3	3	

Finally, use the model to fill in table 2.1f. You may use the number line below to confirm the Frog's location.

Table 2.1f

Directed Length of Jump	Number of Jumps	Frog's Location
$^-3$	$^-6$	
$^-3$	$^-5$	
$^-3$	$^-4$	
$^-3$	$^-3$	
$^-3$	$^-2$	
$^-3$	$^-1$	
$^-3$	0	0
$^-3$	1	$^-3$
$^-3$	2	$^-6$
$^-3$	3	

PROBLEM

Compute the following products. As you multiply, visualize the process to verify the accuracy and reasonableness of your answers.

1. $9 \cdot (-7)$ 2. $-4\,(-8)$ 3 $(-10 \cdot 13) - 10 \cdot 13$ 4. $11 * -5$

SUMMARY (What I learned in this section)

Name:____________________________ **Date:**________ **Period:**_______

SECTION 2.2 AREA MODEL FOR MULTIPLICATION

VOCABULARY

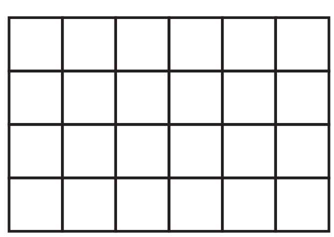
DEFINITION	EXAMPLE
Linear Model:	
Commutative Property of Multiplication:	
Partial Product:	
Distributive Property:	

Big Idea: How do we use the area model to represent multiplication?

EXPLORATION 1: AREA MODEL FOR MULTIPLICATION

A bird refuge is in the shape of a rectangle 4 miles long and 6 miles wide. Draw a visual representation of this refuge on grid paper, using 1 centimeter = 1 mile, and use it to determine the area of this rectangle. Explain how you use the grid to compute the area. Multiply 4 miles by 6 miles using the traditional algorithm only after you have an answer using the visual representation.

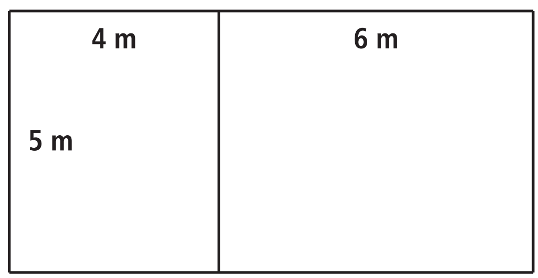
To multiply 4 by 6, consider the picture of the rectangle below. The area of a rectangle is the number of units, or 1×1, squares that it takes to cover the figure with no overlaps and no gaps. What is the area of the rectangle below assuming that each square in the grid has area 1 square unit?



The area is _____ square units. The dimensions of this rectangle are 4 units in width and 6 units in length. Sometimes the dimensions are abbreviated and used to describe the rectangle. The rectangle is 4 units wide and 6 units long: there are 4 rows and 6 columns. The area can be computed by summing the areas of the columns: $4 + 4 + 4 + 4 + 4 + 4 = 4 \cdot 6 =$ ______. We can also think of this area as the sum of the area of the rows: $6 + 6 + 6 + 6 = 6 \cdot 4 =$ ______. The rectangle is called a 4 by 6, or a 6 by 4, rectangle because the area is computed as the product $4 \cdot 6 = 6 \cdot 4 =$ ______. We call this relationship the **commutative property of multiplication**.

EXAMPLE 1:

The Elliots are constructing a small building that is one room wide and two rooms long. Each room is five meters wide. The front room is 4 meters long, and the back room is 6 meters long. The floor plan below shows the layout.



1. What is the floor space of each room? ____________

2. What is the floor space of the building? ___________

3. How are the areas of the two rooms related to the area of the building?

EXAMPLE 2:

Now suppose the dimensions of the Elliots' building have not been decided yet. We need a formula for the areas. Call the width of the building n feet, and the lengths of rooms 1 and 2, k and m feet, respectively. Draw and label the floor plan of the Elliots' building.

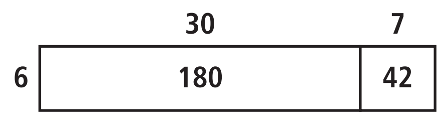
a) What is the floor space of each room? Room 1: _________ Room 2: _________

b) What is the floor space of the building? ___________

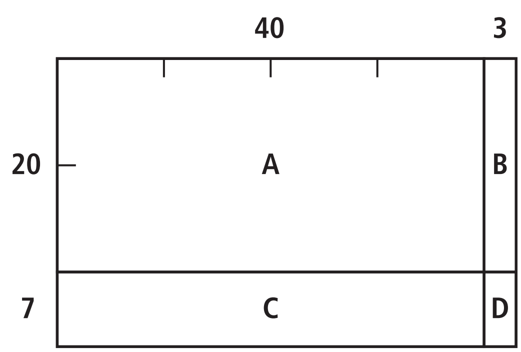
Using Example 2, write a rule for this product: n(k + m) = ____________________. This rule is called the **distributive property**.

EXPLORATION 2: DISTRIBUTIVE PROPERTY

You have already learned to multiply two-digit and three-digit numbers. Now you can use the area model and the distributive property to explore this process carefully. Begin by modeling the product of a one-digit number and a two-digit number. To multiply $6 \cdot 37$, use place value to write the product $6 \cdot 37$ as $6(30+7)$. By the distributive property, $6 \cdot 37 = 6(30 + 7) = 6 \cdot 30 + 6 \cdot 7 = 180 + 42 =$ ________.



Visualize the product of 27 x 43 as area with the picture below:



Area of A = $20 \cdot 40 = 800$;
Area of B = $20 \cdot 3 = 60$;
Area of C = $7 \cdot 40 = 280$;
Area of D = $7 \cdot 3 = 21$. The total area is $800 + 60 + 280 + 21 = 1161$.

You can extend the same process to multiply 27 by 43 using the distributive property, or in the more traditional vertical format.

$$\begin{aligned} 27 \cdot 43 &= (20 + 7)(40 + 3) \\ &= 20(40 + 3) + 7(40 + 3) \\ &= 20 \cdot 40 + 20 \cdot 3 + 7 \cdot 40 + 7 \cdot 3 \\ &= \_\_\_ + \_\_\_ + \_\_\_ + \_\_\_ = \_\_\_\_ \end{aligned}$$

$$\begin{array}{r} 27 \\ \times \; 43 \\ \hline 21 \\ 60 \\ 280 \\ + \; 800 \\ \hline \end{array}$$

PROBLEMS:

1. The Luis family is constructing a rectangular building that is one room wide and two rooms long. Each room is 8 feet wide. The front room is 9 feet long, and the back room is 7 feet long. Create the floor plan that shows this situation.

a) What is the floor space of each room? ____________

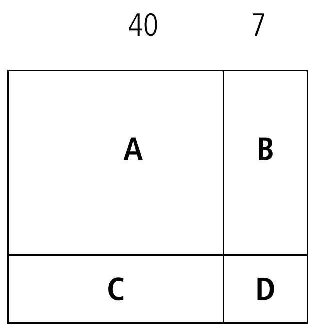
b) What is the floor space of the building? ___________

c) How are the areas of the two rooms related to the area of the building?

2. Use the area model and the distributive property to compute the following products. Indicate the area of each interior part in your model. The first one is started for you.

a. (47) (29) = __________

b. (12) (31) = __________



c. (25) (60) = __________

d. (134) (21) = __________

e. (436) (107) = __________

3. Draw a picture to represent the following products:

   a. $(3x)(2y)$

   b. $6xy$

SUMMARY (What I learned in this section)

Name:____________________________ Date:________ Period:_______

SECTION 2.3 LINEAR MODEL FOR DIVISION

VOCABULARY

DEFINITION	EXAMPLE
Divisor:	
Quotient:	
Dividend:	
Factor:	
Missing Factor Model:	
Remainder:	

Big Idea: How do we use the linear model to perform division?

EXPLORATION 1: BIG WATER BOTTLE

A 32-ounce bottle of water needs to be evenly distributed among 5 students. How many whole number of ounces of water will each student get, and how much water will be left over?

Begin by drawing a number line to show the number of ounces in the big water bottle. Use the Linear Model by skip counting by the number of students in order to distribute the water evenly.

EXPLORATION 1: CONTINUED

a. Each student will receive _________ ounces of water (a whole number).

b. There will be _________ ounces of water remaining

c. Notice when using the remainder, the solution is 32 = _______ · _______ + _______

EXPLORATION 2: MR. GARZA'S CANDY

Mr. Garza has 20 pieces of candy. He wants to divide the candy equally among 6 children. How should he distribute the candy?

One way to distribute the candy is to think of this process in steps. In step 1, give each child 1 piece of candy. This means Mr. Garza has $20 - 6 = 14$ pieces of candy left. In step 2, Mr. Garza gives each child a second piece of candy. He now has $14 - 6 = 8$ pieces of candy left. In step 3, Mr. Garza gives each child a third piece of candy. He now has $8 - 6 = 2$ pieces of candy left. He can no longer give an equal number of pieces to each of the 6 children, so he stops. It took 3 steps to equally distribute as many pieces of candy as Mr. Garza could. That means each child received 3 candies. Write this as $20 = 3 \cdot 6 + 2$. Picture this as a linear model by skip counting to divide 20 by 6, which corresponds to the counting 3 skips of length 6: $3 \cdot 6 = 18$, 2 units short of 20.

In division, the problem involves the dividend and the divisor, and the task is to compute the quotient. In the linear model, the dividend is the total length. There are two possible cases:

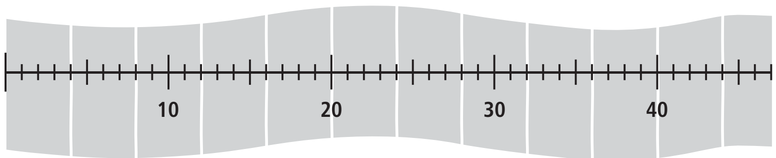
(1) Know the length of each jump and call it the divisor. Find the quotient, which in this case is the number of jumps that equal the total length.

(2) Know the number of jumps and call it the divisor. Find the quotient, which in this case is the length of each jump.

In multiplication, the problem starts with the length of each jump and the number of jumps. The answer is the accumulated length of all the jumps. Again, division is the reverse of the multiplication process.

EXAMPLE 1:

Robin has 47 feet of ribbon on a roll. She wants to cut this roll of ribbon into 4-foot strips for decorations. How many 4-foot strips of ribbon can she make? How much ribbon will be left over, if any?



Notice when using the remainder, the solution is 47 = _______ · _______ + _______

PROBLEMS:

1. Evaluate the following, and write the associated multiplication fact. Use the linear model or long division, if needed. The first one is done for you. (Hint: Think Fact Families!)

   a. 56 ÷ 7 = <u>8 because 8 · 7 = 56</u>

   b. 72 ÷ 12 = ______________________

   c. 238 ÷ 7 = ______________________

   d. 192 ÷ 8 = ______________________

2. Write the associated multiplication fact, making the remainder as small as possible. The first one is done for you.

a. $53 \div 6 =$ 8 R 5 because $53 = 6 \cdot 8 + 5$

b. $84 \div 9 =$ ________________

c. $39 \div 7 =$ ____________________

d. $87 \div 14 =$ _______________

e. $104 \div 33 =$ ________________

f. $456 \div 9 =$ ________________

3. Rachel, Sarah, and Sophia are texting their friends during summer vacation. If together they sent 2,112 messages, and all 3 girls sent the same number of texts, how many texts did each girl send?

4. There are 16 children in Mrs. Marques' math camp class. One day she comes to class with freshly baked cookies to share. She gives each student an equal number of cookies and discovers that she has 9 left over. She knows she started with less than 5-dozen cookies.

   a. What is the largest number of cookies Mrs. Marques could have had originally?

   b. How many cookies would she have given each student in that case?

   c. What other possible solutions could this problem have?

5. Rewrite the following using three different symbols for division:

a. 15 divided by 7 ______________________________

b. 22 divided by *x* ______________________________

c. *y* divided by 25 ______________________________

SUMMARY (What I learned in this section)

MULTIPLYING AND DIVIDING 2

Name:___________________________ Date:_______ Period:______

SECTION 2.4 THE DIVISION ALGORITHM

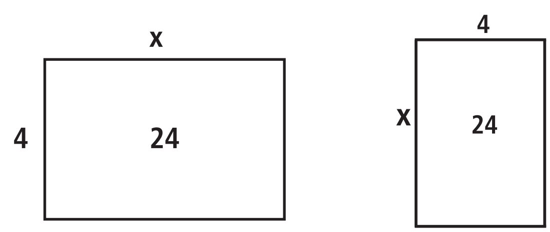
VOCABULARY

DEFINITION	EXAMPLE
Division:	
Algorithm:	

Big Idea: How do we divide using the area model? What is the division algorithm?

EXPLORATION 1: THE DIVISION ALGORITHM

Another way of thinking of division is by using the area model. This is similar to the missing factor model. To divide 24 by 4, draw a length of 4 and ask what the width x of the rectangle must equal to have a total area of 24. What you are doing is looking for the missing factor x, so that: $24 = 4 \cdot x$, and $24 = x \cdot 4$. This is an example of the commutative property of multiplication.

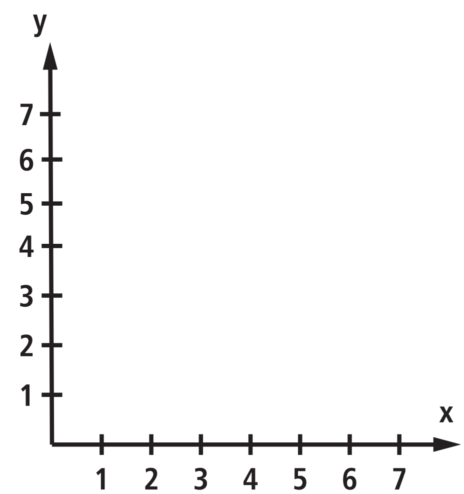


You know that division is the reverse operation for multiplication, just as subtraction is the reverse operation for addition. What do we mean by this? Begin with the number 12. Add 3 to get 15. To undo the addition, you need to subtract 3 from 15 and return to the original number 12. Similarly, in the example above, you found the number 6. Multiply by 4 to obtain 24. That is, $24 = 6 \cdot 4$. To undo this multiplication, divide 24 by 4 and return to the start 6 because $24 \div 4 = 6$.

EXAMPLE 1:

Using the Area Model, what is $20 \div 3$?

To solve, let's look at Quadrant I on the Coordinate Plane.



Since we're dividing by 3, mark off a length of 3 on the y-axis and begin shading a rectangle of width 1 along the x-axis.

a. Notice that at mark 1 on the x-axis, you have a rectangle with an area of _____ square units. This shows that $3 \cdot 1 = 3$, and you would need 17 more squares to reach 20.

b. Shade to the second mark along the x-axis. What is the area of your next rectangle?
____

c. Write the associated multiplication fact if you stopped at the second mark.

Continue shading along the *x*-axis to determine the biggest rectangle you can form with an area less than or equal to 20 square units.

d. Write the dimensions (side lengths) of your rectangle: ______________________

e. How many extra squares would you need to shade to reach 20? __________

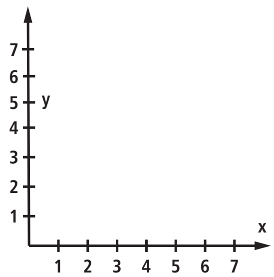
f. Write the associated multiplication fact, making the remainder as small as possible.

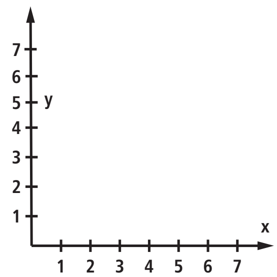
____

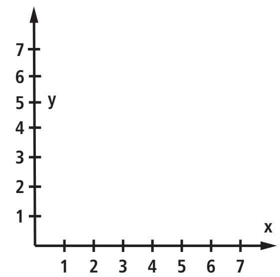
PROBLEMS:

1. Draw the area model for each of the following then use the division algorithm to compute.

a. $15 \div 4$ b. $41 \div 6$ c. $28 \div 5$







2. Given the problem $15 \div 3$, show why the area and linear models for division give the same results.

3. Randyn is making snack bags for her class field trip. She has 791 treats to share evenly among 22 snack bags. Estimate to find about how many treats should be placed in each bag so that she uses up as many of the treats as possible.

4. Destiny is planning a fiesta for her family and friends. There will be 74 people attending, and Destiny wants to place 5 or 6 people at each table. Draw a diagram of the tables and number of people seated at each that satisfy these conditions.

SUMMARY (What I learned in this section)

MULTIPLYING AND DIVIDING 2

Name:____________________________ **Date:**________ **Period:**_______

SECTION 2.5 LONG DIVISION

VOCABULARY

DEFINITION	EXAMPLE
Scaffolding:	

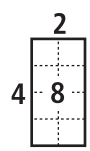
Big Idea: How do we model long division using scaffolding?

EXPLORATION 1: THE DIVISION ALGORITHM

We have seen how closely related multiplication and division are. For example, we know $8 \div 4 = 2$ because $4 \times 2 = 8$. Also recall that in the long division form, the multiplication fact is rewritten as

$$\begin{array}{r} 2 \\ 4\overline{)8} \end{array}$$

The area model looks like this:

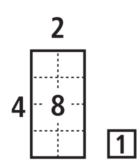


We have the **dividend** 8 "under" the **quotient** 2, and the **divisor** 4 is to the left of the dividend.

By changing the dividend to 9, our problem becomes $9 \div 4$. Because $8 \div 4 = 2$, we see that $9 \div 4$ must be more than 2. In the long division form we have,

$$\begin{array}{r} 2 \\ 4\overline{)9} \\ \underline{-8} \\ 1 \end{array}$$

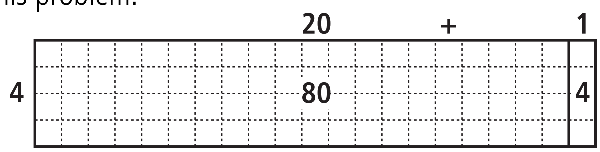
The area model looks like this:



The quotient is 2, and the remainder is 1.

Now consider the problem, 84 ÷ 4. We know that 80 ÷ 4 is 20 and 4 ÷ 4 is 1. Putting these together shows 84 ÷ 4 = 21.

Here is the area model for this problem:



OR

$$\begin{array}{r} 1 \\ 20 \\ 4\overline{)\,84} \\ \underline{-80} \\ 4 \\ \underline{-4} \\ 0 \end{array}$$

This is called the **scaffolding** method because the different partial quotients are first computed and stacked, then combined, much like a scaffold is used in constructing a building.

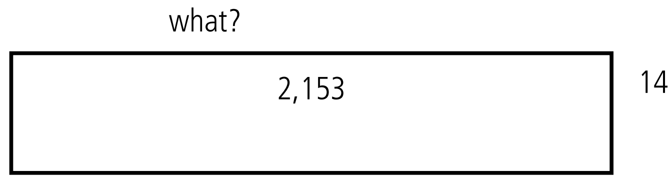
PROBLEMS:

1. Use the scaffolding method to compute the following quotients. You may sketch a picture of the corresponding area model if it helps.

   a. 52 ÷ 4 b. 960 ÷ 6 c. 2,175 ÷ 25

2. In dividing, we know that it is more common to start with the largest place value to determine the quotient and then gradually include the smaller place values. Let’s try a problem using that method. In the space below work 2153 ÷ 14.

Another way to think about this problem is to consider the related multiplication statement. Because the division problem is, "What does 2,153 ÷ 14 equal?" the related multiplication statement reads, "What times 14 equals 2,153?"



3. The area of Robert's dorm room is 96 square feet. The room is 8 feet long. What is the width?

4. Compute the following quotients and remainders. Check your answer with a visual method. Identify the dividend, quotient, divisor, and remainder.

   a. 265 ÷ 2

   Dividend is________.

   Divisor is _________.

   Quotient is ________.

   Remainder is ________.

   b. 99 ÷ 15

   Dividend is ________.

   Divisor is _______.

   Quotient is ________.

   Remainder is ________.

5. Kayla is going to rent some movies for a slumber party. Each movie rental costs $3 per night. If Kayla has $29 to spend, how many movies can she rent?

SUMMARY (What I learned in this section)

MULTIPLYING AND DIVIDING 2

Name:_____________________________ Date:________ Period:_______

CHAPTER 2: SPIRAL REVIEW

1. Evaluate $n + 7$ for each value of n:

   a. $n = 12$, ____________ b. $n = 4$, ____________ c. $n = 11$, ____________

   d. $n = 3$, ____________ e. $n = 86$, ____________ f. $n = 100$, ____________

2. Write an expression to represent each event:

   a. Barb's science report has 4 more pages than Kaylie's. If K represents the number of pages in Kaylie's report, write an expression to represent Barb's report.
   ____

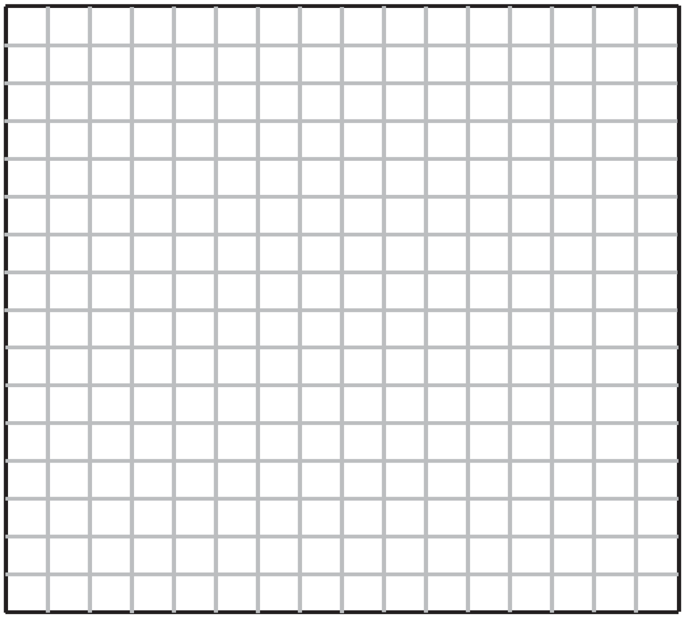
   b. Let s represent the number of students in math camp. Write an expression for the number of students that must be in each of 7 equal math camp groups.
   ____

   c. Juan is 3 years older than Antonio. If a represents Antonio's age, what expression represents Juan's age?____

   d. Let g represent the number of girls in the candy store. If each girl is buying 5 pieces of candy, write an expression that represents the total number of candies the girls will buy.
   ____

3. Write a story problem for the following expression, $t - 3$. Remember to tell us what t represents.

4. On the grid paper below, draw a coordinate plane and label the following parts: Quadrant I, II, III, IV, *x*-axis, *y*-axis, and origin.



5. Use the area model for multiplication to find the product of (412) (43).

6. Write an integer to represent each action:
   a. a $35 deposit ______________________

   b. a loss of 10 yards ______________________

   c. losing 16 pounds ______________________

   d. finding $20 ______________________

   e. ascending 14 km ______________________

   f. 6 new messages ______________________

g. a $10 withdrawal ______________________

h. diving 126 feet ______________________

i. temperature rises 10° ______________________

7. Godfrey has 13 pieces of bubble gum. He bought 5 packages containing 24 pieces each. How many pieces of bubble gum does he have now?____

Godfrey now has _________________ pieces of bubble gum.

8. Fill in the table by multiplying each number by 10, 100, and 1,000:

Number	· 10	· 100	· 1000
82			
27			
105			
340			
1,234			

9. Josh is going to tile the walkway from his back door to his garden bench. The path is 2 feet wide and 15 feet long. He plans to buy square-foot tiles that cost $1.99. How many tiles does he need to buy? About how much will be the total cost?

Josh needs to buy _________ tiles. He will spend about ____________.

10. The area of Josh's rectangular garden is 204 square feet. If the length is 12 feet, what is the width?

FACTORS AND MULTIPLES 3

Name:___________________________ **Date:**_______ **Period:**______

SECTION 3.1 FACTORS, MULTIPLES, PRIMES, AND COMPOSITES

VOCABULARY

DEFINITION	EXAMPLE
Divisible:	
Factor:	
Multiple:	
Prime:	
Composite:	
Multiplicative Identity:	
Square numbers:	

Big Idea: How do you find factors and multiples of a number? How do you determine if a number is prime or composite?

EXPLORATION 1: THE POSSIBLE RECTANGLE MODEL

You will use graph paper for this activity and the Possible Rectangles Chart.

1. For each positive integer n from 1 to 30, make as many rectangles with integer side lengths as you can that have area equal to n square units. Count a rectangle only once if the factors are the same. For example, 3 x 4 and 4 x 3 will be counted only once.

2. Organize the data in the following Possible Rectangles Table. In the first column, you see given values for the positive integer *n*. In the second column, write the number of rectangles possible with area *n*. In the third column, list all the possible dimensions of the rectangles. In the fourth column, list all the possible lengths of sides of the rectangles, in increasing order. For example, we have filled in the results for the value of $n = 4$ on the table.
3. What do you notice in the table so far?
4. Continue the table for *n* from 31 to 50.
5. Looking at the extended table, do the patterns continue?
6. Looking at a given number *n*, what do you notice about the numbers in the last column for this value of *n*?
7. What do we call the numbers in the last column in relation to *n*? For each rectangle, the dimensions form a factor pair, such as 3 and 6 for $n = 18$. If you put all the factors in the last column in order, such as 1, 2, 3, 4, 6, 12 for $n = 12$, how do the factor pairs line up?
8. What do you notice about the number 1? Find any other numbers that have this same property, if possible.
9. Circle the values of *n* that generate only one rectangle.
   How many factors does each of these have?
   How would you describe the circled numbers, excluding 1?
10. Use a different color pen or marker to box the values of *n* that have an odd number of positive divisors.

Notice that all of the factors in our table are positive. Generally, when talking about *factors*, we just mean the positive factors. The numbers that have only two positive factors play a special role in mathematics and have a special name.

Possible Rectangles Table

n	Number of Possible Rectangles	Possible Rectangle Dimensions	Possible Side Lengths (in increasing order)
1			
2			
3			
4	2	1×4, 2×2	1, 2, 4
5			
6			

Possible Rectangles Table (continued)

n	Number of Possible Rectangles	Possible Rectangle Dimensions	Possible Side Lengths (in increasing order)
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			
26			

Possible Rectangles Table (continued)

n	Number of Possible Rectangles	Possible Rectangle Dimensions	Possible Side Lengths (in increasing order)
27			
28			
29			
30			
31			
32			
33			
34			
35			
36			
37			
38			
39			
40			
41			
42			
43			
44			
45			
46			

Possible Rectangles Table (continued)

n	Number of Possible Rectangles	Possible Rectangle Dimensions	Possible Side Lengths (in increasing order)
47			
48			
49			
50			

EXPLORATION 2: SIEVE OF ERATOSTHENES

Exploration 2 is based on an ancient method attributed to a famous Greek mathematician, Eratosthenes of Cyrene. The process involves letting certain kind of numbers pass through the sieve and leaving other kind of numbers in the sieve. Try the Exploration, and see for yourself.

1. Use the grid of the first 100 natural numbers on the next page.
2. Mark out the number 1. We will see why in the next section.
3. Using a colored pencil or marker, circle the number 2, and then mark out every remaining multiple of 2 until you have gone through the whole list. What is a mathematical term for the marked out numbers? ______________________
4. Go back to the beginning of the grid and with a different colored pencil or marker, circle the first number that is not marked out and not circled. Then, mark out all remaining multiples of that number. Notice that some numbers are crossed out by two different colored pencils.
5. Repeat this process until you have gone all the way through the list of numbers and they are now either crossed out or circled.
6. Make a new ordered list of all the circled numbers. What do these numbers have in common? How is this list of numbers related to patterns from the possible rectangle activity?

____

____

7. You might have noticed that in the third round, some of the multiples of 3 were already crossed out in the second round. Find 3 such numbers. Why did this happen?

____

____

____

____

Sieve of Eratosthenes

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

EXPLORATION 3

In small groups or individually, determine whether the following numbers are prime or composite. Try to devise as many timesaving strategies as you can, so you don't have to check every integer between 1 and the target number.

a. 51 ___ b. 67 ___ c. 81 ___

d. 99 ___ e. 113 ___ f. 123 ___

g. 171 ___ h. 131 ___ i. 323 ___

1. What are some strategies for finding all the factors or divisors of a given positive number *n*?

____

____

____

EXPLORATION 4: DIVISIBILITY RULES

Use the Sieve of Eratosthenes to explore the following:

1. What pattern do you notice about numbers that are multiples of 2? Make a conjecture for a rule to determine whether or not a number is divisible by 2.

2. What pattern do you notice about numbers that are multiples of 5? Make a conjecture for a rule to determine whether or not a number is divisible by 5.

3. What pattern do you notice about numbers that are multiples of 10? Make a conjecture for a rule to determine whether or not a number is divisible by 10.

4. What pattern do you notice about numbers that are multiples of 3? Make a conjecture for a rule to determine whether or not a number is divisible by 3.

5. What pattern do you notice about numbers that are multiples of 9? Make a conjecture for a rule to determine whether or not a number is divisible by 9.

6. What pattern do you notice about numbers that are multiples of 6? Make a conjecture for a rule to determine whether or not a number is divisible by 6.

Summarize your findings in the table below.

A number is divisible by….	If…
2	
3	
4	
5	
6	
9	
10	

EXPLORATION 5: POSSIBLE RECTANGLE CHART

Refer back to your table of possible rectangles for this activity.

1. Is it possible to pair up all the positive factors of 36? Explain.

____

____

2. Find all the other numbers between 1 and 100 with an odd number of factors. What might these numbers be called?

____

____

3. On the number line below, locate and plot the numbers (square numbers) associated with square shape.

PROBLEMS:

1. Tell if the number is prime or composite, and justify your response.

   a. 119

   ____

   b. 171

   ____

   c. 127

   ____

2. Is 9 a factor of 112? Explain why or why not.____

   Equivalently, is 112 a multiple of 9? ____

3. Is 18 a factor of 144? Explain why or why not. ______________________

   Equivalently, is 144 a multiple of 18? Explain why or why not.______________________

   Use a T-Chart of factor pairs to solve this problem.

4. Is the number 105 divisible by 15? ____________ Explain why or why not.

   ____

   Is every number divisible by 15 also divisible by 3 and 5? ____________ Explain your answer.

   ____

5. Use factor T-charts to find all the factors of the following numbers.

   a. 51 b. 108 c. 405

6. Miguel wants to cut an 84-foot length of rope into pieces of equal lengths with no rope left over. List all the possible lengths he could use.

7. List the first ten multiples of the following numbers.

   a. 5 ______________________________

   b. 14 ______________________________

   c. 17 ______________________________

8. Consider the factors and multiples of the number 12.

   a. The factors of 12 are: ______________________________.

   b. The multiples of 12 are:
   ______________________________.

   c. Is it possible to list all the multiples of 12? Explain.

9. In your own words, explain the difference between factors and multiples. Use complete sentences.

____

____

____

____

____

____

SUMMARY (What I learned in this section)

FACTORS AND MULTIPLES 3

Name:____________________________ **Date:**________ **Period:**_______

SECTION 3.2 EXPONENTS AND ORDER OF OPERATIONS

VOCABULARY

DEFINITION	EXAMPLE
Exponential Notation:	
Power or exponent:	
Base:	
Order of Operations:	

Big Idea: How do you use order of operations to compute numerical expressions?

EXPLORATION 1: EXPONENTIAL GROWTH

Your uncle is asking you what you would prefer for your birthday gift, and gives you a choice of $50 every year for your birthday or $1 for this birthday and double the amount every year thereafter.

Fill in the table below to compare the outcome of the two choices.

Year	1	2	3	4	5	6	7	8	9	10	11	12	13
Choice 1: $50 per year	$50	$50	$50										
Choice 2: $1 doubling each year	$1	$2	$4										

a. After 13 years, what is the total amount you would have received if you selected Choice 1? ______________________________

b. After 13 years, what is the total amount you would have received if you selected Choice 2? ______________________________

EXPLORATION 2: REPEATED MULTIPLICATION

Consider the problem 2 + 2 + 2 + 2 + 2 = 10. We know that we can write this problem more simply using multiplication as $2 \cdot 5 = 10$. Now consider $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 =$ ______. Just as multiplication takes the place of repeated addition, we can use a base and an exponent to indicate repeated multiplication, where the base is the repeated factor and the exponent, or power, tells how many times the base is multiplied by itself, as illustrated below.

EXPONENTIAL FORM	EXPANDED FORM	STANDARD FORM
base → 2^5 ← exponent	$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	= 32

EXPONENTIAL FORM	EXPANDED FORM	STANDARD FORM
7^2		
	$6 \cdot 6 \cdot 6$	
		243
	$1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$	
10^6		
	$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$	
		25
	$11 \cdot 11 \cdot 11$	
2^8		
	8	

EXPLORATION 3: MULTIPLICATION OF POWERS

By using the definition of exponential notation and multiplication, we see that:

$$3^4 \cdot 3^6 = (3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3) = 3^{10} = 3^{4+6}.$$

Compute the following products, showing all your work.

a. $3^2 \cdot 3^3$ ______________ b. $3^3 \cdot 3^2$ ______________

c. $2^4 \cdot 2^2$ _____________ d. $10^3 \cdot 10^5$ ______________

What pattern do you observe when multiplying numbers in exponential form with the same base? Explain.

____

____

EXPLORATION 4: SPECIAL CASES

What do 4^1 and 4^0 equal?

We note that $4 \cdot 4 = 4^2 = 4^{1+1} = 4^1 \cdot 4^1$, so 4^1 must be the same as 4. We can use the same process for any number x: $x \cdot x = x^2 = x^{1+1} = x^1 \cdot x^1$, so $x^1 = x$.

What does 4^0 equal? Because $4 \cdot 4^0 = 4^1 \cdot 4^0 = 4^{1+0} = 4^1 = 4 = 4 \cdot 1$, we see that multiplying by 4^0 is the same as multiplying by the number 1. We, therefore, assume that for any positive integer n, $n^0 = 1$.

Consider the number 4638, which we read as four thousand six hundred thirty eight. Using place value and our notation of exponents, we can rewrite 4638 using expanded notation in the following way:

$$4 \cdot 1000 + 6 \cdot 100 + 3 \cdot 10 + 8 \cdot 1 = 4 \cdot 10^3 + 6 \cdot 10^2 + 3 \cdot 10^1 + 8 \cdot 10^0$$

Start with the expression $4 \cdot 10^3 + 6 \cdot 10^2 + 3 \cdot 10^1 + 8 \cdot 10^0$, or in calculator notation, $4 \times 10^3 + 6 \times 10^2 + 3 \times 10^1 + 8 \times 10^0$. In what order can we perform the calculations in this expression so the sum equals 4638?

To summarize, any number to the first power is equal to the base and any non-zero number raised to the power of zero is equal to 1. Every time!

Rewrite each of the following in standard form.

1. $4^1 =$ ______________________
2. $7^0 =$ ______________________
3. $n^1 =$ ______________________
4. $5^1 + 4^0 =$ ________________
5. $x^1 - y^0 =$ _________________

EXPLORATION 5: ORDER OF OPERATIONS

Compute the following, showing all your work.

$$20 - 10 \div 2 + 3^3 - 9$$

____

____

____

If possible, check your work on a calculator. (To enter 3^3 on a calculator, we enter 3^3). Would you be surprised to know that the answer is not 9? The order that a calculator uses is called the Order of Operations.

We summarize below the order in which mathematical operations are performed:

Order of Operations

Compute the numbers inside the parentheses or grouping symbols.

Compute any exponential expressions.

Multiply or divide as they occur from left to right.

Add or subtract as they occur from left to right.

Why do these two problems have different solutions?

a. $7 \cdot 8 - 6 \div 2$

b. $7 \cdot (8 - 6) \div 2$

Let's try our first problem again using the correct order. Carry out one operation for each step and write your result. Continue this process until you are able to find the one number that equals the numerical expression given. Add more steps if you need them.

$20 - 10 \div 2 + 3^3 - 9$ = ______________________

= ______________________

= ______________________

= ______________________

= ______________________

EXPLORATION 6: MORE ORDER OF OPERATIONS

PEMDAS or GEMS

Use the table as a guide and work your problem in the right-hand column.

PEMDAS		GEMS		$4 + 2^3 \cdot 3 - (17\text{-}5) \cdot 2 + (17\text{-}5) \div 2$
P	Parenthesis ( )	**G**	Grouping ( ), [ ], { }, | |	
E	Exponents x^y	**E**	Exponents x^y	
MD	Multiply **OR** Divide, Whichever comes first from left to right	**M(D)**	Multiply **OR** Divide, Whichever comes first from left to right	
AS	Add **OR** Subtract, Whichever comes first from left to right	**S(A)**	Add **OR** Subtract, Whichever comes first from left to right	

PROBLEMS:

1. Rewrite each of the following in exponential form.

   a. $18 \cdot 18 \cdot 18 \cdot 18 \cdot 18 \cdot 18 \cdot 18 \cdot 18 =$ ____________________

   b. $(4 \cdot 4 \cdot 4 \cdot 4) + (87 \cdot 87) =$ ____________________________________

   c. $5 \cdot 5 \cdot 5 \cdot 5 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 =$ _______________________________

   d. $n \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n =$ ___________________________

   e. $5 =$ _____________

   f. $s \cdot s \cdot s \cdot b \cdot b =$ ______________________

2. Expand and compute the answer of the following. Tell which expression is greater or if they are equal. For example,

   4^3 or $3^4 = 4 \cdot 4 \cdot 4$ or $3 \cdot 3 \cdot 3 \cdot 3$. Then $\underline{64 < 81}$

   a. 2^3 or $4^2 =$ _______________________________. Then ______________________

   b. 5^3 or $3^5 =$ _______________________________. Then ______________________

   c. 1^6 or $4^0 =$ _______________________________. Then ______________________

3. Evaluate the following expressions by expanding then writing your solution.

   a. $4^2 + 8 =$ ____________________________________ = ________________

   b. $3^3 + 5^2 =$ __________________________________ = ________________

   c. $3^4 + 4^3 =$ __________________________________ = ________________

4. Evaluate the following numerical expressions using Order of Operations:

   a. $7 + (5 - 2)^3 - 16 \div 2$

   b. $(81 \div 9)^2 \cdot (27 - 24)$

   c. $24 \div 8 \cdot 7 - 1^4$

d. $9 - 5 \div (8 - 3) \cdot 2 + 4$

e. $16 - 3(8 - 3)^2 \div 5$

f. $7 + (6 \cdot 5^2 - 4^2)$

5. Escherichia coli bacteria are more commonly known as E. Coli. A scientist places one of the living bacteria in a petri dish. The number of bacteria in the dish doubles each hour.

   a. How many bacteria are in the dish after 1 hour? ________________

   b. How many bacteria are in the dish after 3 hours? ________________

   c. How many bacteria are in the dish after 5 hours? ________________

   d. How many bacteria are in the dish after *n* hours? ________________

SUMMARY (What I learned in this section)

FACTORS AND MULTIPLES 3

Name:____________________________ **Date:**________ **Period:**_______

SECTION 3.3 PRIME FACTORIZATION

VOCABULARY

DEFINITION	EXAMPLE
Prime Factorization:	
Perfect Square:	
Perfect Cube:	
Fundamental Theorem of Arithmetic:	

Big Idea: How do you find the prime factorization of numbers?

EXPLORATION 1: FACTORING A NUMBER AS PRODUCT OF PRIMES

Consider the number 60. What number pairs can we multiply together to get 60? Make a list of the factor pairs.

____

____

Now, let's choose just one of the factor pairs. For example, $4 \cdot 15 = 60$. Are either of these numbers prime? No! Let's find the factors *of the factors*.

$4 = 2 \cdot 2$ and $15 = 3 \cdot 5$

Looking at $2 \cdot 2$ and $3 \cdot 5$, we see that all of these numbers are prime. (It may be helpful to refer back to your Sieve of Eratosthenes that you created in Section 3.1.) Therefore, the prime factorization of 60 is $2 \cdot 2 \cdot 3 \cdot 5$.

It's fun to factor a number to the product of prime numbers, and easy to check your work. Find the product of $2 \cdot 2 \cdot 3 \cdot 5$, and verify that it is 60.

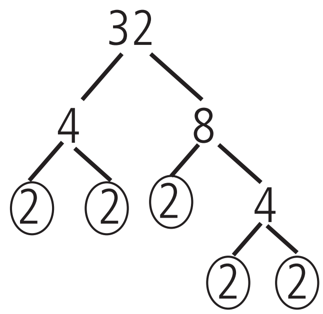
If you had chosen a different factor pair to begin with, would your final answer be different? Pick another factor pair for 60 and check.

Now try to write 24 as a product of prime numbers: ______________________________

Don't forget to check your work!

EXPLORATION 2: FACTOR TREES

A useful way to organize your work is through a **tree diagram**, or **factor tree**.



In the example above, 32 is factored to 4 · 8 in the first branch of the factor tree. Since neither 4 nor 8 are prime, the branches will continue until we reach only prime numbers. One reason we are so interested in prime numbers is that they are the building blocks of the integers. In the previous section, we learned that a prime number is a positive integer *greater than 1* that can be written as a product of only two positive integers. Keep in mind that 1 is *not* a prime number. Therefore, it should not be included in your factor tree.

Find the prime factorization of the following numbers using a factor tree.

a. 48 b. 108 c. 2,500

EXPLORATION 3: PRIME FACTORIZATION CHART

Use any method you choose to prime factor the integers 1 – 100. Write the prime factorization in exponential form, as shown in the examples already done for you.

1	Not prime or composite!	21		41		61		81	3^4
2	2	22		42		62		82	
3	3	23		43		63		83	
4		24		44		64		84	
5		25		45		65		85	
6	$2 \cdot 3$	26		46		66		86	
7		27		47		67		87	
8		28		48		68		88	
9		29		49		69		89	
10		30		50		70		90	
11		31		51		71		91	
12		32		52		72	$2^3 \cdot 3^2$	92	
13		33		53		73		93	
14		34		54		74		94	
15		35		55		75		95	
16		36		56		76		96	
17		37		57		77		97	
18		38		58		78		98	
19		39		59		79		99	
20		40		60		80		100	

EXPLORATION 4: PERFECT CUBES

A **perfect cube** is an integer *n* that can be written in the form $n = k^3$, where *k* is an integer. Some examples of perfect cubes are

$$0^3 = 0,\ 1^3 = 1,\ 2^3 = 8,\ 3^3 = 27,\ 4^3 = 64,\ \ldots.$$

How can you use the prime factors of a number to determine whether it is a perfect cube?

____

____

PROBLEMS:

1. Determine as efficiently as possible whether each of the following numbers is prime or composite. Prove your answer with a factor pair if you believe the number is composite.

   a. 51 ____________________

   b. 235 ____________________

   c. 159 ____________________

   d. 119 ____________________

   e. 31 ____________________

   f. 790 ____________________

2. Write the prime factorizations of 12 and 24. Looking at your factorizations, explain what the answers have in common and what are their main differences.

   ____

   ____

   ____

3. Write the prime factorizations of 8 and 49 in exponential form. Looking at your factorizations, explain what the answers have in common and what are their main differences.

   ____

   ____

   ____

SUMMARY (What I learned in this section)

FACTORS AND MULTIPLES 3

Name:___________________________ Date:_______ Period:______

SECTION 3.4 Common Factors and the GCF

VOCABULARY

DEFINITION	EXAMPLE
Common Factor:	
Greatest Common Factor:	
Relatively Prime:	

Big Idea: How do you find the greatest common factor of two or more numbers?

EXPLORATION 1: FERNANDO AND THE FROG-JUMPING CONTEST

Pick three different colors to represent the 1-frog, 2-frog, and 3-frog in the activity that follows.

To prepare for a frog-jumping contest, Fernando decided to train a group of his fellow frogs. Each frog was trained to jump a certain length along a number line starting at 0. He trained a 1-frog to jump a distance of 1 unit in each hop. He also trained a 2-frog to jump 2 units, a 3-frog to jump 3 units and so on. The frogs always start at the zero point on the number line. Now Fernando wants to know which frogs will land on certain locations on the number line. (Use the number line provided to answer the following questions.)

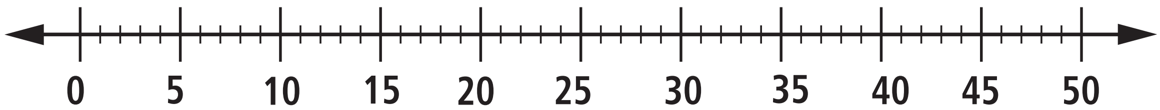
1. Which of his frogs will land on both the locations 24 and 36? ______________________________

2. Which is the longest jumping frog that will land on both 24 and 36? Explain why this answer makes sense.

____

____

3. What is the longest jumping frog that will land on both 20 and 32? ______________________



4. What is the longest jumping frog that will land on both 24 and 25? ______________________.
Use the number line above to explain.

EXPLORATION 2: IDENTICAL STRINGS

Suppose we have different lengths of two types of string. The cotton string is 120 inches long, and the nylon string is 72 inches long. Determine every possible integer length both of the strings can be cut so that each piece is the same length. Make a list of all the possible common lengths. What is the longest common length possible? In this Exploration, each piece must be cut into a positive integer length with no fractions and no string left.

The longest common length possible is: ______________________

EXAMPLE 1:

There are several different ways to calculate the GCF of two numbers. Here is one way that reinforces the term **G**reatest **C**ommon **F**actor. Find the GCF of 15 and 25 using a Factor T-Chart listing all of the factor pairs.

15		25	

List all the factors 15 and 25 have in common. ______________________________

What is the greatest factor these two numbers have in common? ____________________

EXAMPLE 2:

Find the GCF of 27 and 32.

27		32	

List all the factors 27 and 32 have in common. ______________________________

What is the greatest factor these two numbers have in common? ____________________

These integers are examples of relatively prime numbers. In your own words, write a definition of relatively prime numbers.

____

____

____

From the definition above, the numbers 27 and 32 are relatively prime. Notice that neither 27 nor 32 are prime numbers. If we consider two prime numbers like 3 and 7, what is their GCF? Check a few more examples. Make a generalization about the GCF of any two prime numbers.

____

____

____

EXAMPLE 3:

Although Factor T-Charts are effective, they are not as efficient with larger numbers. Consider the integers 108 and 168 using the process of first finding the factors, then the common factors, and finally the GCF.

First, list all the factors of 108. Then, list all the factors of 168. Make sure you have 12 factors for 108 and 16 factors for 168. There are many factors to find. If you do not have them all listed, go back and find them. Next, determine all the factors the two numbers have in common.

Find the GCF of 108 and 168.

108 | 168

List all the factors 108 and 168 have in common. ______________________________

What is the greatest factor these two numbers have in common? ____________________

As you discovered, this method for finding the GCF works well. However, the more factors the numbers have, the more time it takes to make the list of factors for each number. Fortunately, prime factorization makes finding the GCF of two numbers easier

Here is an example of the efficiency of prime factorization. Start by finding the prime factors of 108 and 168. Use the space below to make your factor trees.

108 | 168

Now list the prime factors of each number. Your job will be easier if you list the prime factors in order from least to greatest and stack like numbers in a column so that it is clear which prime factors the two numbers have in common.

108 = ______________________________

168 = ______________________________

Common Prime Factors: ________________________

The product of the common prime factors is the GCF. What is the GCF of 108 and 168? _____

Try using prime factorization to find the GCF of 343 and 140. Use the space below for your factor trees.

List the prime factorization for the two integers and look for common prime factors.

343 = ___________________________________

140 = ___________________________________

Common Prime Factors = ____________________________________

GCF of 343 and 140 = __________

PROBLEMS:

1. Find the Greatest Common Factor (GCF) of each pair of numbers.
   a. GCF of 12 and 15 = __________

   b. GCF of 45 and 48 = __________

   c. GCF of 48 and 57 = __________

d. GCF of 80 and 64 = ____________

e. GCF of 50 and 35 = ____________

f. GCF of 24 and 18 = ____________

2. In each part of this exercise, the prime factorizations of two numbers are given. First, use the prime factorizations to find the GCF of the two numbers then, compute (find the value of) the two numbers from their prime factors.

a. $2 \cdot 3 \cdot 7$

$2 \cdot 3 \cdot 3 \cdot 5 \cdot 7$

GCF = ________

Values: ______________

b. $2^3 \cdot 3^2$

$2 \cdot 3^3$

GCF = ________

Values: ______________

c. $5 \cdot 7^2$

$2 \cdot 7^3$

GCF = ________

Values: ______________

3. For each pair of integers below, find the GCF of the two integers using prime factorization.

a. 24 and 62 b. 115 and 225 c. 79 and 83

GCF of 24 and 62 = ________ GCF of 115 and 225 = ________ GCF of 79 and 83 = ________

4. Sarah has 36 red beads, 45 yellow beads, and 63 green beads. She wants to separate them into identical groups to make bracelets for her friends. What is the greatest number of bracelets she can make so that each bracelet has the same number of each color of bead and there are no beads left over?

Sarah can make __________ bracelets.

Each bracelet will have ______ red, ______ yellow, and _______ green beads.

5. LiiLii has 40 carnations, 32 roses, and 24 sprigs of baby's breath. She is making flower arrangements to set at the tables in her diner. What is the greatest number of vases she can fill with matching arrangements?

LiiLii can make _______ arrangements.

Each arrangement will have ______ carnations, ______ roses, and _______ sprigs of baby's breath.

6. There are 12 girls and 18 boys in Ms. Girardeau's math class. What is the greatest number of identical groups Ms. Girardeau can make so that no children are left out?

Ms. Girardeau can make ________ groups with ______ boys and _____ girls in each group.

7. Make a list of any keywords or phrases that indicate finding the Greatest Common Factor.

SUMMARY (What I learned in this section)

Name:__________________________ **Date:**_______ **Period:**______

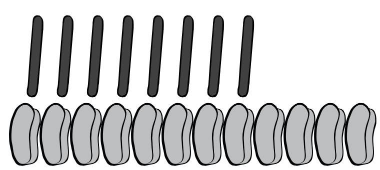
SECTION 3.5 COMMON MULTIPLES AND THE LCM

VOCABULARY

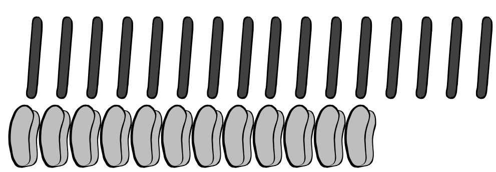
DEFINITION	EXAMPLE
Common Multiple:	
Least Common Multiple:	

Big Idea: How do you find the least common multiple of two or more numbers?

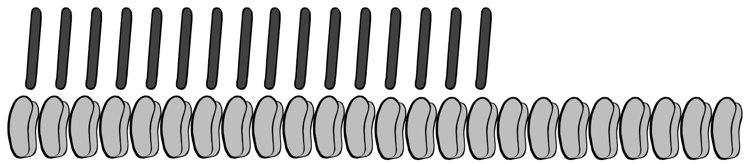
Have you noticed that hot dogs often come in packages of eight, and hot dog buns come in packages of twelve? When people plan to cook hot dogs, they tend to buy one package of hot dogs and one package of buns. But if they do this, they are left with four extra buns.



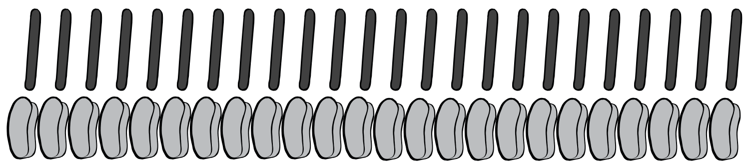
Some people who pay for the extra hot dog buns don't want to waste them. What can they do? They could buy another package of eight hot dogs:



But now there are four extra hot dogs without buns. If they buy more buns:



There are eight buns without hot dogs, even more extra buns than the first time. Will this process ever end? Try buying one more package of hot dogs:



Aha! We have finally reached a point where we have exactly the same number of hot dogs and buns. Of course, in order to get there, the consumers had to buy two packages of buns and three packages of hot dogs. Maybe they can freeze the rest.

What happened mathematically with the hot dogs and buns? One way to organize the number of hot dogs and the number of buns is to create a table. Fill in the missing cells on the table.

# Packages	1	2	3	4	5	6		
Hot dogs	8	16						
Buns	12							

Because you want to buy the least amount possible, you look through your table to find the smallest number they each have in common. Then determine how many packages of hot dogs and hot dog buns you will need to buy in order to have the same amount of each.

The smallest number they each have in common is ___________. Therefore, you should buy ______ packages of hot dogs and ________ packages of hot dog buns.

Notice that 48 and 72 are also common multiples of 8 and 12 but not the least.

EXPLORATION 2: COMMERCIAL TIME

Radio station KISS broadcasts a commercial every 22 minutes. WILD broadcasts a commercial every 12 minutes. If the two stations broadcast their commercials at 3:20 , when is the next time their commercials will air at the same time?

To solve this problem we begin by listing the multiples of 22 and 12. You can organize this on a table. List the first 10 multiples in the table below.

12	24	36							
22	44								

Do you see a common multiple? Sometimes it is necessary to continue skip counting. Since 12 is the smaller number, let's extend our list of multiples further.

What are the next two multiples of 12? ______________________________

By now, you should see a common multiple. What is the LCM of 12 and 22? __________

Looking back at our original problem, we see that we not only need to find the LCM, but also the next time the commercials will be playing simultaneously. Remember, they aired together last at 3:20. Use the LCM to determine your answer.

The commercials will air together again at ______________.

EXAMPLE 1:

As we found with the GCF, there are several different ways to calculate the LCM of two numbers. Once again, we'll see that prime factorization can make this process simpler.

Let's try finding the LCM of 54 and 63 using prime factorization. Begin by using a factor tree to find the prime factorization of 54 and 63.

54

63

List the prime factors of each integer from least to greatest, stacking the common prime factors:

54: ______________________________

63: ______________________________

Now look at the method for finding the LCM of numbers using their prime factorization. The factors of each are as follows: 54: $3 \cdot 3 \cdot 3 \cdot 2$ and 63: $3 \cdot 3 \cdot 7$. By examining the two sets of prime factors, you can see that a common multiple must include 2, 3, and 7. However, $2 \cdot 3 \cdot 7 = 42$ is not a multiple of 54 or 63. Because 54 has three factors of 3 and 63 has two factors of 3, to include both numbers, use three factors of 3. Why won't two factors of 3 be enough? Now multiply the factors 2, 3^3, and 7.

$$2 \cdot 3^3 \cdot 7 = \_\_\_\_\_\_\_\_$$

This is the smallest integer that contains all of the building blocks, or prime factors, in both sets of prime factors. Therefore, __________ is the LCM of 54 and 63.

Now find the LCM of each pair of numbers below using the prime factorization method:

a. 36 and 42

36 **42**

Prime Factor Lists

36: ______________________________

42: ______________________________

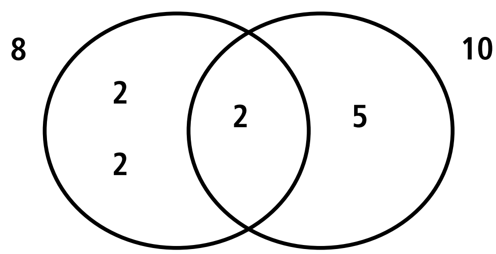
LCM: ______________

b. 17 and 18

c. 13 and 52

EXPLORATION 3: FINDING THE LCM WITH A VENN DIAGRAM

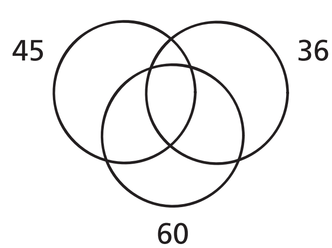
Another approach to solving for the LCM is to look at the Venn diagram, as you did in Section 3.4, when you worked with GCFs. Examine the prime factors of 6 and 8. The Venn diagram includes the prime factors for each number in the respective circles. Note the common factor of 2 in the overlapping part of the circles. 2 is the GCF. What must the LCM be?



You will find that the Venn diagram is more practical when the numbers are larger and there are three numbers or more. For example, to find the least common multiple of 45, 36, and 60, write all the prime factors and notice which factors are common.

$$45 = 3^2 \cdot 5$$
$$36 = 2^2 \cdot 3^2$$
$$60 = 2^2 \cdot 3 \cdot 5$$

Use the Venn diagram to represent this information.



After you have separated the factors into different regions in the Venn diagram by multiplying all the numbers in the circles and their intersections you will have the LCM of 45, 36, and 60. Show your work below.

The LCM of 45, 36, and 60 is ____________________.

PROBLEMS:

1. Find the Least Common Multiple of each pair of numbers by listing multiples of the two numbers until you find the first multiple common to both lists.

   a. 6 and 9, LCM = _____________

   b. 15 and 18, LCM = _____________

   c. 8 and 12, LCM = _____________

2. In each part of this exercise, the prime factorizations of two numbers are given. First, use the prime factorizations to find the LCM of the two numbers then, compute (find the value of) the two numbers from their prime factors.

a.	b.	c.
$2 \cdot 3 \cdot 5$	$2^2 \cdot 3^2$	$2 \cdot 5 \cdot 7^2$
$2 \cdot 5 \cdot 7 \cdot 9$	$2 \cdot 3^3$	$2 \cdot 7^3$
LCM = ________	LCM = ________	LCM = ________
Values: _______________	Values: _______________	Values: ______________

3. For each pair of integers below, find the LCM of the two integers using prime factorization.

a. 24 and 62 b. 115 and 225 c. 79 and 83

LCM = ________ LCM = ________ LCM = ________

4. Brian and Lydia are running laps around the track. Brian can run one lap in 7 minutes, and Lydia can run a lap in 3 minutes. If they start together, how many minutes will pass before they are crossing the start line together again?

Brian and Lydia will cross the starting line together again in __________ minutes.

5. The red shuttle bus at Texas State University stops at every fifth building on University Drive. The blue shuttle bus stops at every fourth building on this route. If they leave the first building together, what building will they both stop at next?

The red and blue shuttle buses will both stop at the _______________ building next.

6. In a long line of goody bags, Shannon is placing a special prize in every fourth goody bag. Colton is placing a different prize in every fifth bag. Peyton is placing another prize in every sixth bag. What is the first bag in this line of goody bags that will have all three prizes?

The __________ bag is the first goody bag that will have all three prizes.

7. On its opening night, Hilary's Hula House is giving every tenth customer a free ice cream cone, every twelfth customer a free slice of pie, and every twentieth customer a free Hula skirt. Which customer will be the first to receive all three free items?

The ___________ customer will receive all three free items.

8. Make a list of the keywords or phrases you notice with LCM word problems.

SUMMARY (What I learned in this section)

FACTORS AND MULTIPLES 3

Name:____________________________ Date:________ Period:_______

CHAPTER 3: SPIRAL REVIEW

1. Tell whether each number in the table is divisible by 2, 3, 4, 5, 6, 9, and 10 by placing a check in the appropriate columns.

	2	3	4	5	6	9	10
90							
416							
1,036							
636							
450							
912							
252							
345							
209							
810							

2. Use the Order of Operations to solve the following problems:

a. $7 \cdot 3^2 + (54 \div 9)$ b. $(8^2 \div 8) + (9 \cdot 5)$

3. Write an algebraic expression that states "5 less than a number".

4. Factor each number to a product of its prime numbers. Write your final answer in exponential form. If a number is prime, write the word "Prime" in the box.

260	123	507
109	300	125
351	105	2,525

5. A banquet hall purchased 196 cloth napkins for place settings. There are eight tables in the executive section. Can each location have the same number of place settings? _______________ Explain why or why not and state the possible number of place settings at each table.

____

____

6. Use exponents to write the numbers three different ways:

81			
64			
16			

7. In each problem below, determine if you should use the GCF or LCM to find your answer. Write "GCF" or "LCM" in the margin beside each problem, and then solve.

   a. Sammie made a row of tiles 84 centimeters long. Darla's row was 96 centimeters long. What is the largest size tile they might have been using if all tiles were the same length? Explain.

      The tiles were ____________cm.

   b. Taylor is buying cookies and juice boxes for her class. Cookies are sold by the dozen, and juice boxes come 8 in a pack. What is the least number of packages of each that Taylor can buy in order to have one cookie for each juice box? Show how you arrived at your answer.

      Taylor will buy _______ packs of cookies and ______ packs of juice boxes.

   c. The trolley stops at every third corner in downtown San Antonio. On the same route, the city bus stops at every eighth corner. If the trolley and the bus leave at the same time following the same route, what is the first corner that they will both stop at?

      The trolley and bus will both stop at the ________ corner.

d. Mr. Carter's gym classes will be competing in groups to finish a set of obstacle courses. There are a total of 84 girls and 126 boys in all of his classes. What is the largest number of identical groups that Mr. Carter can form so that everyone participates and nobody is left out?

He can form ________ groups with ______ girls and _____boys in each.

e. A frog and a grasshopper start jumping at the same time from the same location. The frog jumps 16 cm, and the grasshopper jumps 12 cm with each leap. What is the first spot they will both land on?

They will both land at _______ cm.

f. Sasha keeps her room tidy by performing certain tasks on a specific schedule. For example, she vacuums her carpet every three days, dusts her furniture every other day, and cleans the windows once a week. If she did all three tasks on the same day, May 1st, what is the next date she will again perform all three on the same day?

Sasha will do all three tasks again on ______________.

8. Use the calendar to answer the following questions:

Sun	Mon	Tues	Wed	Thurs	Fri	Sat
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

Connor enrolled in summer classes at his local library. He signed up to attend Culinary Skills for Kids every three days, Bird Watching every 4 days, and Painting classes every 5 days. Now, he's worried that he may have signed up for too many classes! His first day of classes was the 1st of the month.

a. On which days does he only have the Culinary Skills class?

____

b. On which days does he only have the Bird Watching class?

____

c. On which days do both the Culinary Skills and Bird Watching classes meet?

____

d. On which day(s) do both the Bird Watching and Painting Classes meet?

____

e. How many times do the Culinary Skills and Painting classes meet on the same day? ________

What dates? _______________

f. Do all three classes ever meet on the same day? _________ Explain.

____

9. Find the LCM and GCF of these numbers: 12, 20, 30.

LCM = _______________ GCF = _______________

10. Compare and contrast the factors and multiples of an integer. Choose a number to use as an example, but also write an explanation using complete sentences.

FRACTIONS 4

Name:___________________________ Date:________ Period:_______

SECTION 4.1 MODELS FOR FRACTIONS

VOCABULARY

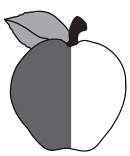
DEFINITION	EXAMPLE
Numerator:	
Denominator:	
Equivalent:	
Equivalent Fractions:	
Discrete Model:	
Simplifying:	
Simplest Form:	

Big Idea: How do you model fractions?

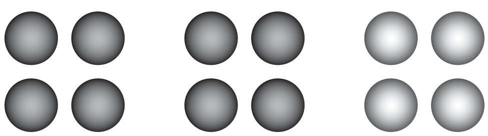
EXPLORATION 1: MODELS FOR FRACTIONS

In this section you will examine models that represent fractions. Think of some examples you have seen that represent fractions and draw them in the space below.

There are two basic ways to represent fractions: as part of a whole or part of a group. The picture below illustrates one apple divided in half. The shaded part represents half of the apple.

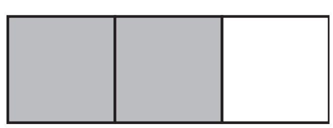


Next, divide a number of marbles into three equal groups. Two-thirds of the marbles is two of these three equal groups



Now think of a fraction as part of a pan of brownies. This is called the **area** or **brownie model** for a fraction. Pictures representing the numbers one-half and two-thirds can look like these:

and



We write the two fractions mathematically as:

One-half = _______ Two-thirds = _________

We use two numbers in writing a fraction, the **numerator** and the **denominator**. The numerator is above the denominator; it indicates how many parts are shaded. "Numerator" comes from the same root as "number." It counts the number of parts. The denominator tells how many parts the whole or group is divided into. "Denominator" comes from the same root as "name." It names the parts that the whole is divided into, like halves or fourths.

Use the area model to draw the fractions below. Draw the whole as a rectangle because it is easier to divide into equal pieces. For each fraction, identify the numerator and denominator. Then write the fraction mathematically.

a. Three-fourths

b. Two-fifths

c. One-fifth

d. Three-tenths

EXPLORATION 2: PAPER FOLDING ACTIVITY

You will need several sheets of paper for this activity. Each sheet represents one whole.

Step 1: Fold the paper to represent the number $\frac{1}{2}$. Write $\frac{1}{2}$ on each of the two parts of the folded paper. Is there more than one way to represent $\frac{1}{2}$?

____

Step 2: Use a new sheet to create $\frac{1}{4}$. How many parts equal to $\frac{1}{4}$ are there in the whole sheet of paper? What fraction represents three of these parts? What represents two parts of the paper?

____

Step 3: Use a new sheet of paper to make a folded piece that has eight equal parts. Identify and make a list of as many fractions involving the denominator 8 as you can. Which of these fractions represent the same fractional part as the fractions in Step 1 and 2 with different denominators?

____

Step 4: Fold this same sheet of paper once more to make sixteenths. How many total times did you fold the paper?

____

EXPLORATION 3: EQUIVALENT FRACTIONS

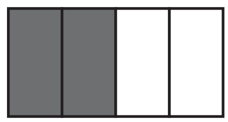
It is possible for two fractions with different numerators and denominators to represent the same amount? Consider the following example:

$\frac{1}{2} =$ 

Notice that we can draw a larger picture, but it still represents the same part of the whole.

$\frac{1}{2} =$ 

If we divide a whole into 4 equal parts, 2 of the 4 parts can represent the fraction;

$\frac{2}{4} =$ 

or we can draw it as a larger picture.

$\frac{2}{4} =$ 

Are these both a representation of $\frac{2}{4}$? They do not look the same. So what is the difference?

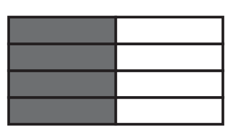
Shading 2 equal parts out of 4 is equivalent to shading 1 part out of 2. This means the fractions $\frac{1}{2}$ and $\frac{2}{4}$ are **equivalent**. Another way to show that the fraction $\frac{1}{2}$ is equivalent to the fraction $\frac{2}{4}$ is to take the picture representing $\frac{1}{2}$ and draw a horizontal slice as shown below:

$\frac{2}{4} =$ 

The horizontal slice doubles the numerator and also doubles the denominator, the number of parts the whole is divided into.

Suppose we make three horizontal cuts in the original rectangular model for $\frac{1}{2}$ to form equal-sized pieces. What fraction is shaded?

____



Like the example above, the picture represents both $\frac{1}{2}$ and $\frac{4}{8}$. These fractions are equivalent. We write $\frac{1}{2}=\frac{2}{4}=\frac{4}{8}$ because the fractions represent the same part of a whole.

Remember, the "whole" is not just a geometric shape that represents one whole, like a circle or a rectangle, subdivided into equal parts. For example, the "whole" might be a class that has 8 girls and 10 boys. What fraction of the class is female? Male? ____________________

Let's start a chart of equivalent fractions. In each column below, write fractions that are equivalent to each other. Be sure to use the fractions from the paper folding activity as well as any from our class discussions. You can also use a number line and label the corresponding points with equivalent fractions. This may be a useful reference as you work with fractions in the sections that follow. The first column is completed for you.

EQUIVALENT FRACTIONS CHART

$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$
$\frac{2}{4}$						
$\frac{4}{8}$						
$\frac{8}{16}$						

EXAMPLE 1

A class of 12 consists of 4 boys and 8 girls. We know that the fraction of the class that is male can be written $\frac{4}{12}$ and the fraction of the class that is female can be written $\frac{8}{12}$. Use equivalent fractions and find another way to express the fraction of the class that is boys as well as the fraction of the class that is girls.

____

Write three additional ways these fractions can be written.

$\frac{4}{12} = \_\_\_ = \_\_\_ = \_\_\_$ $\qquad \frac{8}{12} = \_\_\_ = \_\_\_ = \_\_\_$

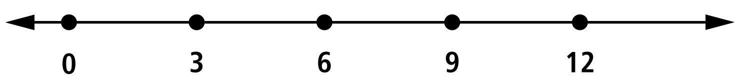
EXAMPLE 2

Sandra buys a pack containing a dozen pencils. She sharpens $\frac{3}{4}$ of the pencils. How many pencils did Sandra sharpen? ____________________

This is an example of the **discrete model** for fractions. Discrete means being made up of individual parts, just like groups of students. In this model, we assume we have a collection or group of *n* objects and subgroup of *m* objects. The fraction $\frac{m}{n}$ represents the part of the whole group that is in the subgroup.

A number line is another way to find equivalent fractions. Let's examine the idea using this problem.

Now that you have divided the dozen pencils equally into 4 groups, you can use a number line and make 4 jumps like this:



Notice that if each jump represents a divided grouping of the 12 pencils, then one jump lands at 3, two jumps at 6, 3 jumps land on 9, and 4 jumps land on 12. $\frac{3}{4}$ of the dozen pencils is equivalent to 3 jumps or 9 pencils.

Draw a discrete model that shows that $\frac{2}{8}$ of the birds are blue.

Write an equivalent fraction for $\frac{2}{8}$ using the smallest number possible as the denominator.

$$\frac{2}{8} = \frac{\quad}{\quad}$$

Denise wants to buy a video game that costs \$60. So far, Denise has raised $\frac{2}{3}$ of the cost. How much money has she raised? How much more money does she need to raise? Use a visual model to explain.

EXPLORATION 4: FINDING EQUIVALENT FRACTIONS

Find three equivalent fractions for each of the fractions below. You may use paper folding or any other model to visually show that your fractions are equivalent fractions.

a. $\frac{1}{2}$ __________ __________ __________

b. $\frac{1}{8}$ __________ __________ __________

c. $\frac{1}{4}$ __________ __________ __________

d. $\frac{2}{5}$ __________ __________ __________

e. $\frac{2}{3}$ __________ __________ __________

Do you see a pattern when two fractions are equivalent? Describe how can you make an equivalent fraction from a given fraction without a model?

____

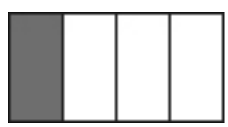
____

In general, we can find equivalent fractions by multiplying the numerator and denominator by the same number. For example,

$$\frac{1}{4} = \frac{2 \cdot 1}{2 \cdot 4} = \frac{1 \cdot 2}{4 \cdot 2} = \frac{2}{8}$$

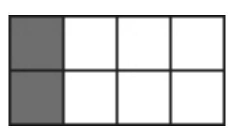
Pictorially,

$$\frac{1}{4} =$$



Multiplying the numerator and denominator by 2 has the effect of doubling the number of slices:

$$\frac{1}{4} = \frac{1 \cdot 2}{4 \cdot 2} = \frac{2}{8} =$$

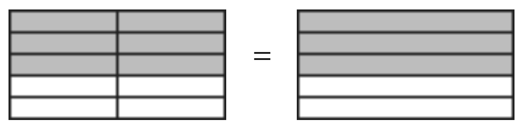


Multiplying the numerator and denominator by the same number changes the number of shaded parts and the total number of parts by the same factor, yielding an equivalent fraction.

EXPLORATION 5: SIMPLIFYING FRACTIONS WITH THE GCF

We have generated equivalent fractions using the area model by dividing a given representation into smaller equal pieces, and converting 1 part out of 4 parts into 2 parts out of 8 parts. $\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16}$ and so on. Notice that in this case, the new denominator is a multiple of the original denominator. Must this always be true?

Many times we will want to find an equivalent fraction with a smaller denominator, if possible. We call this process **simplifying** a fraction. We will do this by using Property 4.1 in reverse. For example, to simplify the fraction $\frac{6}{10}$, we first recognize that $\frac{6}{10} = \frac{2(3)}{2(5)}$. The numerator 6 and the denominator 10 have a common factor, 2. Dividing both the numerator and the denominator by the common factor of 2 produces an equivalent fraction. Using the Equivalent Fractions Property, we see that $\frac{6}{10}$ is equivalent to $\frac{3}{5}$.



$\frac{6}{10} = \frac{3 \cdot 2}{5 \cdot 2}$ OR $\frac{6}{10} = \frac{3}{5}$

So, we have simplified $\frac{6}{10}$ to the form $\frac{3}{5}$. Notice that we are using the **Equivalent Fraction Property**,

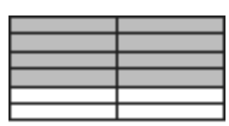
$$\boldsymbol{\frac{a \cdot k}{b \cdot k} = \frac{a}{b}}.$$

A fraction is said to be in **simplest form** if the numerator and denominator have no common factors except 1.

EXAMPLE 3

Write $\frac{8}{12}$ in simplest form.

Look at a rectangular model with $\frac{8}{12}$ shaded in.



Notice that we can also view this fraction as $\frac{4}{6}$.

$\frac{8}{12} = \frac{4 \cdot 2}{6 \cdot 2} = \frac{4}{6}$

However, notice that this fraction can also be written equivalently as $\frac{2}{3}$.

$$\frac{\mathbf{8}}{\mathbf{12}}=\frac{4}{6}=\frac{2\cdot2}{3\cdot2}=\frac{2}{3}$$

Once you find the factors of each numerator and denominator as shown below,

8	
1	8
2	(4)

12	
1	12
2	6
3	(4)

you can identify the greatest common factor, GCF, of the numerator and the denominator.

Use it to simplify the given fraction by applying the equivalent fraction property so that

$$\frac{8}{12}=\frac{2\cdot4}{3\cdot4}=\frac{2}{3}$$

Write the following problems in simplest form, using Factor T-Charts to find the GCF of the numerator and denominator.

1. $\frac{24}{60}$

2. $\frac{15}{45}$

3. $\frac{27}{63}$

4. $\frac{9}{10}$

EXPLORATION 6: SIMPLIFYING FRACTIONS BY PRIME FACTORIZATION

The GCF is useful in finding the simplest equivalent fraction.

We will illustrate the usefulness of prime factorization in also computing the simplest form for a fraction. Examine the fraction $\frac{108}{168}$. Knowing the GCF of 108 and 168, we can write the fraction using the equivalent fraction property as:

$$\frac{108}{168}=\frac{2\cdot2\cdot3\cdot3\cdot3}{2\cdot2\cdot2\cdot3\cdot7}=\frac{(12)9}{(12)14}=\frac{9}{14}$$

or you may use the equivalent fraction property multiple times by simplifying the common factors, 2, 2, and 3 without finding the GCF.

Simplification of fractions is only one application for prime factorization, as we will see.

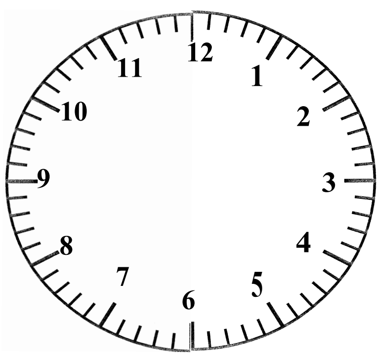
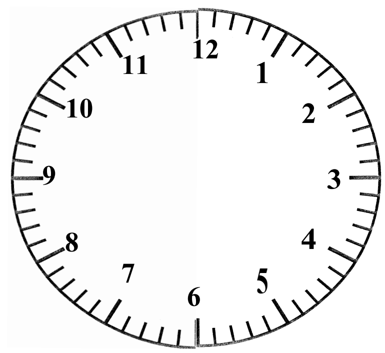
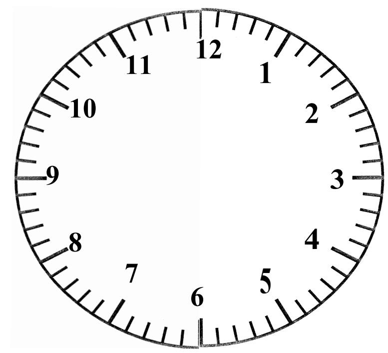
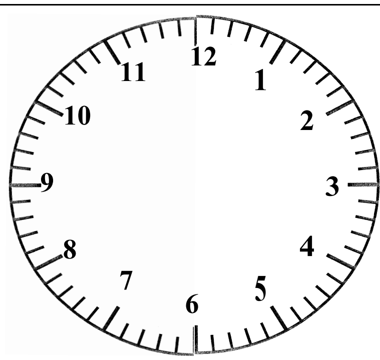
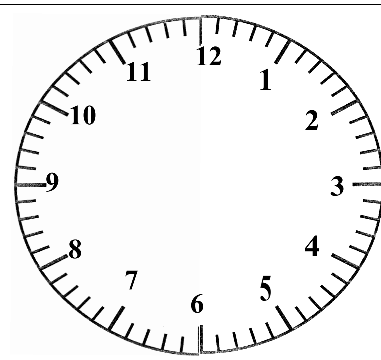
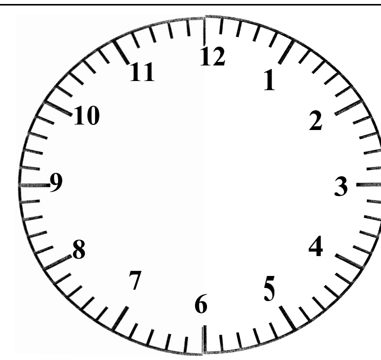
Let's try one together. Consider the fraction $\frac{18}{48}$. Begin by listing the prime factors of the numerator and denominator on the line beside each number.

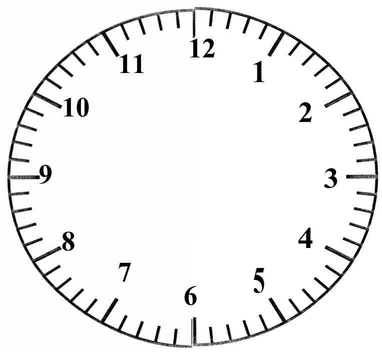
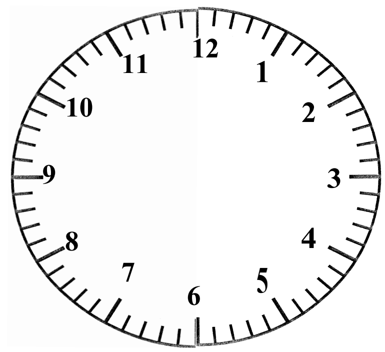
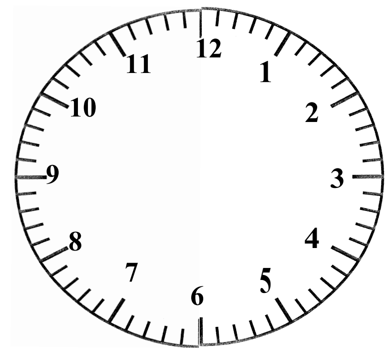
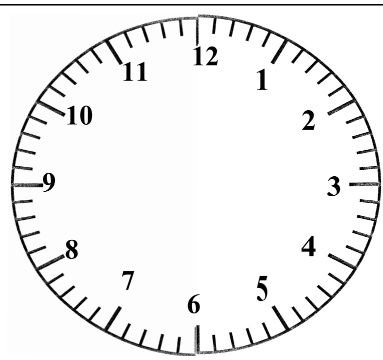
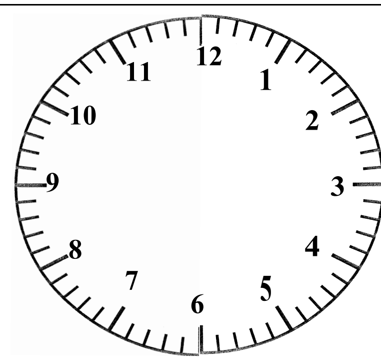
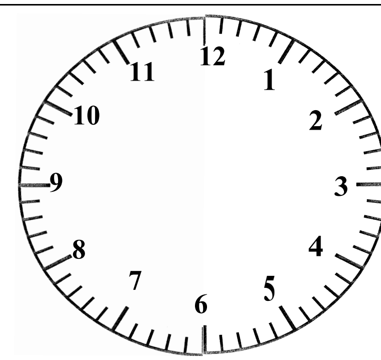
$$\frac{18}{48} = \underline{\hspace{6cm}} = \underline{\hspace{3cm}} = \underline{\hspace{1.5cm}}$$

Circle the prime factors that are common to the numerator and denominator. The GCF is the product of these prime factors. The simplified numerator and denominator are the products of the remaining factors.

EXPLORATION 7: TICK-TOCK FRACTIONS

Let's look at how fractions are used with the concept of time. Remember that 60 minutes equals one hour. On each clock face below, shade the fraction of an hour that is indicated, then complete the statement.

 $\frac{1}{2}$ of an hour = _____ mins	 one quarter of an hour = _____ mins	 three-quarters of an hour = __________ mins
 $\frac{2}{5}$ of an hour = _______ mins	 $\frac{5}{6}$ of an hour = ______ mins	 $\frac{2}{3}$ of an hour = _______ mins

 20 mins = _____ of an hour	 35 mins = ______ of an hour	 18 min = _______ of an hour
 $\frac{7}{12}$ of an hour = _______ mins	 $\frac{1}{3}$ of an hour = _______ mins	 1 min = _______ of an hour

PROBLEMS

1. Simplify the following fractions using prime factorization. Show all your work.

   a. $\frac{12}{48}$ = ______________ = ___________ = __________

   b. $\frac{6}{18}$ = ______________ = ___________ = __________

   c. $\frac{88}{121}$ = _____________ = ____________ = __________

d. $\frac{72}{108}$ = ________ = ________ = ________

e. $\frac{60}{75}$ = ________ = ________ = ________

2. Find four fractions that are equivalent to $\frac{13}{20}$.

____

3. For each of the following fractions, find a common factor in the numerator and denominator. Then simplify.

a. $\frac{45}{63}$ = ________ = ______

b. $\frac{18}{35}$ = ________ = ______

c. $\frac{19}{76}$ = ________ = ______

d. $\frac{17}{51}$ = ________ = ______

4. Mamie practiced shooting hoops for 35 minutes. For what fraction of an hour did she practice?

____

Write this fraction in simplest form. ____________

5. Ricky practiced the Salsa dancing for 42 minutes. For what fraction of an hour did he practice?

Write this fraction in simplest form. ____________

6. In the space below, model the fraction $\frac{1}{2}$. Show how you can divide the model to form equivalent fractions.

SUMMARY (What I learned in this section)

FRACTIONS 4

Name:__________________________ **Date:**_______ **Period:**______

SECTION 4.2 COMPARING AND ORDERING FRACTIONS

VOCABULARY

DEFINITION	EXAMPLE
Common Denominator:	

Big Idea: How do you compare and order fractions?

EXPLORATION 1: LINEAR MODEL FOR FRACTIONS ACTIVITY

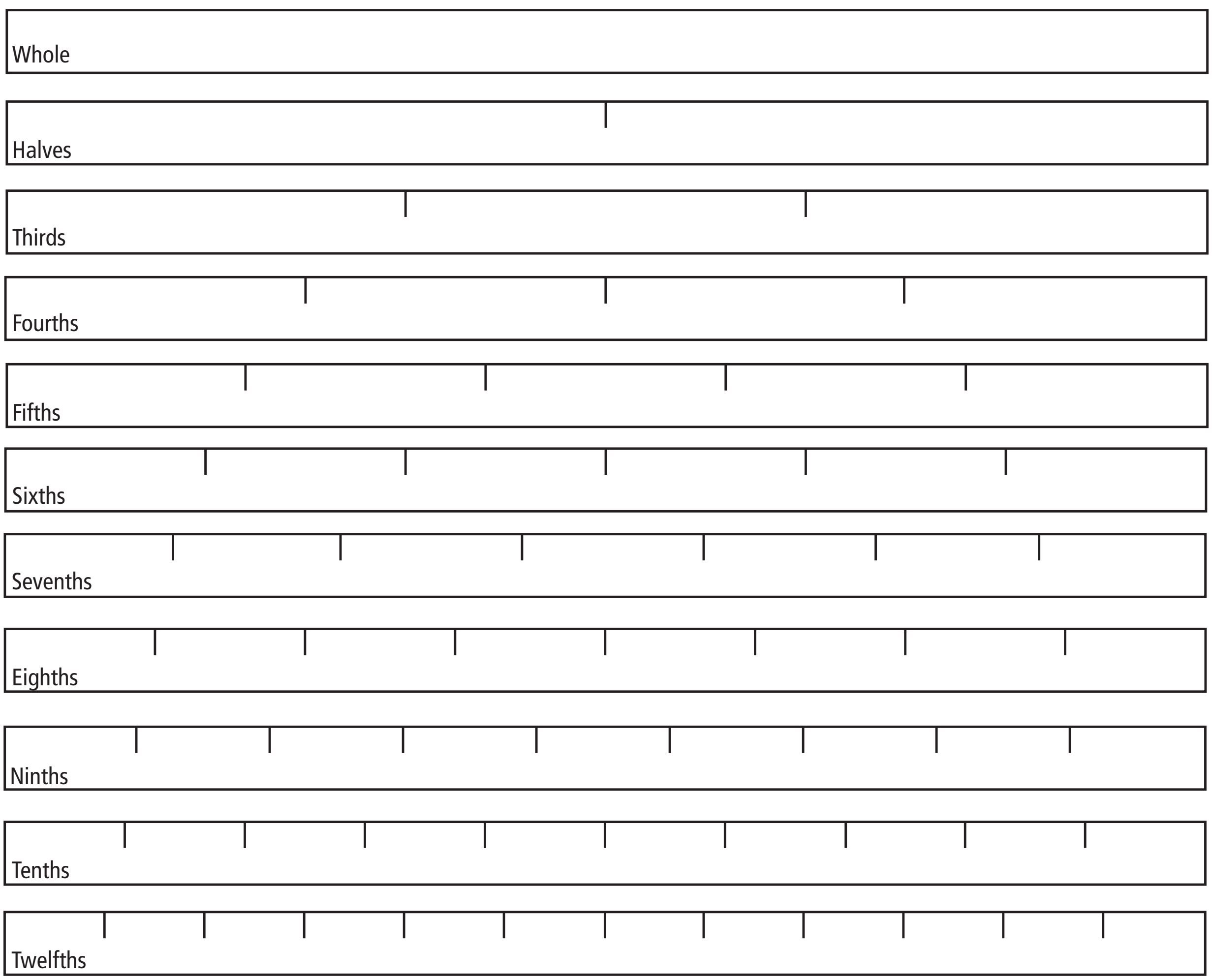
On a number line, each integer corresponds to a point. Recall that there are many other points between each pair of integers on the number line, and each of these points also corresponds to a number. We will now construct a number line from 0 to 1. Use the strip above to create your linear model.

1. Mark the left end point of the strip as 0 and the right end point as 1. You may fold this strip or use another strip to fold and transfer points to this master number line. Fold the strip end to end into two equal parts and mark the crease as the midpoint between 0 and 1. What fraction is this midpoint equivalent to? Label the points on the number line as fractions above the line.
2. Fold the strip again and use the creases to mark and label points on the master number line. Because the strip is now folded into 4 equal parts, we label the first point as $\frac{1}{4}$. Label the other points as $\frac{2}{4}$ and $\frac{3}{4}$. Write the equivalent fraction $\frac{2}{4}$ under $\frac{1}{2}$.
3. Repeat this method by folding the strip again into 8 equal parts, transferring the locations to the master number line and labeling the points with fractions. Use this method to locate, mark and label all the eighths on the number line.
4. Compare the number line with a typical foot ruler or yardstick.

5. Label these fractions on the number line.
   a. $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{5}{5}$ b. $\frac{1}{10}, \frac{2}{10}, \ldots, \frac{9}{10}, \frac{10}{10}$ c. $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}$
   d. $\frac{1}{6}, \ldots, \frac{5}{6}, \frac{6}{6}$ e. $\frac{1}{9}, \ldots, \frac{8}{9}, \frac{9}{9}$
6. Use your new number line to determine which benchmark (0, $\frac{1}{2}$ or 1) each fraction is closest to.
   a. $\frac{4}{12}$ c. $\frac{5}{16}$ e. $\frac{4}{5}$
   b. $\frac{2}{3}$ d. $\frac{8}{12}$ f. $\frac{5}{50}$

The rows of the fraction strips below represent the result of folding the whole into equal parts to represent fractions as we did in the Linear Model Activity. The computer that created it allows the folds to be very accurate. For example, using the Fraction Strips Chart, determine which fraction is greater: $\frac{2}{3}$ or $\frac{3}{8}$.

Fraction Strips Chart



Given two fractions, how can you determine which of them is greater? We can now locate fractions on the number line. The fraction that is to the right of the other is the greater. What problems might arise from this method? For one, its accuracy depends on the quality of the comparative number lines. It becomes harder as the fractions get closer to the same value.

Use the master number line that you constructed or the Fraction Chart to decide which fraction is greater, $\frac{2}{3}$ or $\frac{3}{4}$. Explain your answer. Is there another way to explain which is greater?

EXPLORATION 2: COMMON DENOMINATOR METHOD

The common denominator method compares two fractions by rewriting the given fractions equivalently with the same denominators. Consider the fractions $\frac{2}{5}$ and $\frac{3}{8}$. Which is greater? The first step is to find the least common multiple of the denominators, or LCD (least common denominator). The least common multiple of 5 and 8 is 40. Next, write equivalent fractions for each using 40 as the denominator.

$$\frac{2}{5} = \frac{2 \cdot 8}{5 \cdot 8} = \frac{16}{40} \qquad \frac{3}{8} = \frac{3 \cdot 5}{8 \cdot 5} = \frac{15}{40}$$

When comparing the numerators of the equivalent fractions, it is easier to see which is greater. Let's try some. We see that $\frac{16}{40} > \frac{15}{40}$ and therefore $\frac{2}{5} > \frac{3}{8}$.

Write equivalent fractions to compare which fraction is greater. Use <, > or =.

a. $\frac{3}{5}$ and $\frac{12}{50}$ ____ ☐ ____

b. $\frac{3}{4}$ and $\frac{7}{8}$ ____ ☐ ____

c. $\frac{5}{7}$ and $\frac{7}{10}$ ____ ☐ ____

EXPLORATION 3: ORDERING FRACTIONS

Consider the following problems.

Lorenz has four different wrench sizes given by their diameter measures: $\frac{5}{8}$ inch, $\frac{11}{16}$ inch, $\frac{3}{4}$ inch and $\frac{1}{2}$ inch. Determine the order of the wrenches from largest to smallest.

First, find the LCM of the four denominators to write equivalent fractions. Once you determine the correct order of the fractions, rewrite your answer using the original fractions.

The order of the wrenches will be ______________________________.

Take a look at a similar problem.

Kassandra has 3 pieces of different length ribbons. She has $\frac{7}{8}$ meter of blue ribbon, $\frac{5}{6}$ meter of green ribbon, and $\frac{2}{3}$ meter of yellow ribbon. She needs to use the longest ribbon for a picture frame and the shortest ribbon for a bracelet. The remaining ribbon is going to be used for a bow on a gift. What color ribbon should be used for the picture frame? Bracelet? Bow?

The color for the picture frame is _____________________________________, the bracelet is

____________________________ and the bow is __________________________________.

EXPLORATION 4: ORDERING NEGATIVE FRACTIONS

Draw a number line and locate 0 and -1.

a. Use the number line to locate the following points: $-\frac{1}{2}$, $-\frac{1}{4}$, $-\frac{1}{3}$.

b. Determine which of the three numbers, $-\frac{1}{2}$, $-\frac{1}{4}$, $-\frac{1}{3}$ is greatest.

c. Determine which of the three numbers, $-\frac{1}{2}$, $-\frac{1}{4}$, $-\frac{1}{3}$ is smallest.

Use the space below to construct your number line:

PROBLEMS

1. Write the following fractions in simplest form by first prime factoring the numerators and denominators

a. $\frac{34}{51} =$ ________ = ________

b. $\frac{42}{50} =$ ________ = ________

c. $\frac{72}{108} =$ ________ = ________

d. $\frac{55}{75} =$ ________ = ________

e. $\frac{13}{42} =$ ________ = ________

f. $\frac{123}{222} =$ ________ = ________

2. Which fraction is closer to 1, $\frac{5}{8}$ or $\frac{4}{7}$? Plot on the number line below to justify your answer.

________ is closer to 1 because____

3. Arrange the following fractions in order from least to greatest:

a. $\frac{5}{8}, \frac{4}{5}, \frac{8}{11} =$ ____

b. $\frac{1}{3}, \frac{13}{45}, \frac{2}{5} =$ ____

4. Chuck and his friends are having a frog-jumping contest. Chuck's frog leapt $\frac{5}{6}$ of a yard, Barney's frog leapt $\frac{9}{10}$ of a yard, and Axle's frog leapt $\frac{4}{9}$ of a yard. Write the frog-owners' names in order from first to third place. Explain how you reached your conclusions.

SUMMARY (What I learned in this section)

FRACTIONS 4

Name:__________________________ **Date:**_______ **Period:**______

SECTION 4.3 Unit Fractions, Mixed Fractions, Proper and Improper Fractions

VOCABULARY

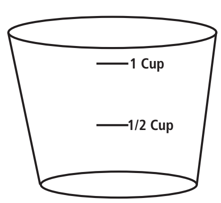
DEFINITION	EXAMPLE
Unit Fraction:	
Mixed Fraction or Mixed Number:	
Improper Fraction:	
Proper Fraction:	

Big Idea: How are mixed fractions and improper fractions related?

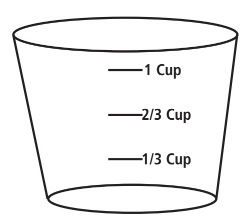
EXPLORATION 1: INTRODUCTION TO MIXED NUMBERS AND IMPROPER FRACTIONS

Recall on the Fraction Chart in Section 4.2 we have $\frac{1}{2}+\frac{1}{2}=1$, $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$.

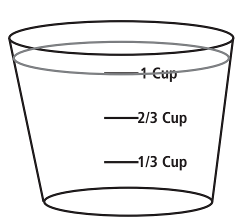
Notice that two halves equal 1

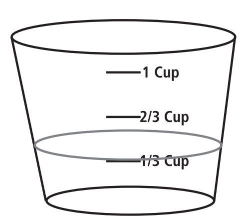


and three-thirds equal 1



What does $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$ equal?





A **unit fraction** always has 1 in the numerator. The denominator is a positive integer.

For example, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and so on are all examples of unit fractions.

Now extend the addition of unit fractions to make another connection to multiplication. You have seen several models that represent the fraction $\frac{3}{5}$. In the area model, $\frac{3}{5}$ represents three $\frac{1}{5}$'s of a whole. This means $\frac{3}{5}$ is the sum of $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}$. In the frog model, this is the same as taking 3 jumps of length $\frac{1}{5}$. That is, $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=\frac{1}{5}\cdot 3=\frac{3}{5}$. This understanding can be extended to all fractions. For example, the fraction $\frac{5}{9}$ is the same as the sum of 5 copies of $\frac{1}{9}$:

$$\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}=\frac{1}{9}\cdot 5=\frac{5}{9}.$$

EXAMPLE 1

Let's say you are going to bake cookies and the recipe calls for $1\frac{3}{4}$ cups of sugar. You look throughout the kitchen and the only measuring cup you can find is the $\frac{1}{4}$ c. measuring cup. How can you use that cup to measure the $1\frac{3}{4}$ cups of sugar you need? Draw a picture and record your answer below.

You will need ________ $\frac{1}{4}$ -cups.

EXAMPLE 2

Look at the mixed fraction $2\frac{1}{4}$. If you have only a quarter-cup measure, describe how you can measure the correct amount with the quarter cup.

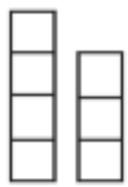
____

____

Did you find $2\frac{1}{4}$ equivalent to $\frac{9}{4}$? In fact, what you have found are two ways to write the same quantity: as a **mixed fraction**, $2\frac{1}{4}$, and as an **improper fraction**, $\frac{9}{4}$. How would you describe improper fractions? Why do you think they are called improper?

$\frac{9}{4}$  $2\frac{1}{4}$

If we have seven quarters, we can think of each quarter as $\frac{1}{4}$ of one dollar. Then we have $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$. Because each of the four quarters equals one dollar, you have 1 dollar and 3 more quarters or $1\frac{3}{4}$ is equal to $\frac{7}{4}$.



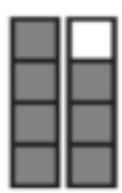
Use a model and repeated addition sentences to find the improper fraction for each of the following mixed fractions:

a. $1\frac{7}{8}$

b. $3\frac{3}{4}$

EXAMPLE 3

Another way to think about $\frac{7}{4}$ is to view this fraction as a division problem, 7 divided by 4. If we divide 7 by 4, we have a quotient of 1 with a remainder of 3. Using the area model, we group 4 of the 7 into a rectangle of dimension 4 by 1 because 4 is the divisor. The remaining 3 fill up $\frac{3}{4}$.

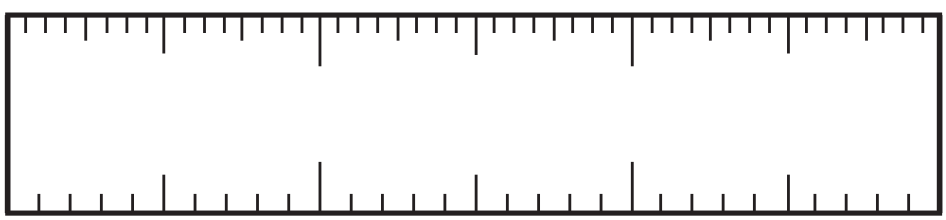
$\mathbf{\frac{7}{4}}$  $\mathbf{1\frac{3}{4}}$

$$\mathbf{\frac{7}{4} = 7 \div 4 = 1\frac{3}{4}}$$

Complete the table below. The first one is done for you.

IMPROPER FRACTION	DIVISION PROBLEM	MIXED FRACTION
$\frac{11}{6}$	$11 \div 6$	$1\frac{5}{6}$
$\frac{53}{12}$		
$\frac{73}{10}$		

EXPLORATION 2: RULER ACTIVITY



Use the above ruler for the following activities.

1. The ruler represents 3 inches. Label the marks representing 1 inch, 2 inches, and 3 inches.
2. How many equal pieces is the top scale divided into? What should we call these pieces?
3. How many equal pieces is the bottom scale divided into? What should we call these pieces?
4. Label the tick marks for each scale.
5. Which tick mark would be equal to a length of $\frac{11}{4}$? Label this mark with an *A*. What is the mixed fraction name for this length?
6. How many $\frac{1}{16}$ inches are in $2\frac{1}{4}$ inches? ________________
7. How many $\frac{1}{8}$ inch are in $2\frac{1}{4}$ inches? ________________
8. Write two other ways to say $\frac{2}{3}$ inches. ________________ ________________

9. Which tick mark on the bottom scale would be equal to a length of $\frac{17}{10}$? Label this mark with a *B*. What is the mixed fraction name for this length? What is the decimal name for this length?

10. Comparing the two scales, which number is greater: $\frac{11}{8}$ inches or $\frac{14}{10}$ inches? What are the mixed fraction names for these lengths?

11. How much greater is $2\frac{1}{4}$ than $1\frac{1}{16}$? ________________

12. If you had been told the above ruler was 6 inches long, how would that change the scale? In other words, how would that have changed the way you marked the tick marks?

PROBLEMS:

1. Rewrite each sum as an improper fraction and as a mixed number.

UNIT FRACTIONS	IMPROPER FRACTION	MIXED NUMBER
$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=$		
$\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=$		
$\frac{1}{2}+\frac{1}{2}=$		
$\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}=$		

2. How many one-sixth cups of sugar are in $1\frac{2}{3}$ cups of sugar?________________

3. Which is greater, $2\frac{5}{8}$ or $\frac{30}{16}$? _____________ Explain your reasoning.

4. Each slice of pizza is $\frac{1}{8}$ of a pizza. You and your friends are to get 3 slices each. If you have 6 pizzas, how many people can be served?

5. Mrs. Miranda cut construction paper into fifths to pass out to her students to make bookmarks. If she passed out $\frac{90}{5}$ how many pieces of construction paper were in the box to begin with?

6. Place zero in the center of the number line below. Number by thirds from $-2\frac{1}{3}$ to $2\frac{1}{3}$.

Place the following numbers on the number line.

a. $-\frac{8}{6}$ b. $\frac{8}{12}$ c. $-\frac{4}{12}$ d. $\frac{5}{3}$

7. Use the number line below to plot points in the following problems.

a. A recipe for pancakes calls for $1\frac{3}{4}$ cups of flour. Locate this point on the number line above. Describe the equivalent improper form for the mixed fraction. What does the numerator represent?

____

b. Jack has 3 identical pans of brownies and decides to divide each pan into 12 equal pieces. How many brownies pieces does he have in all? ____

Because Jack was very hungry, he ate 2 of the pieces. If you assume each brownie pan represents 1 or a whole, express the amount of brownies that remains in terms of the whole and pieces.

c. If Jack takes half of the uneaten brownies to a party, what quantity will he take? ____________

Using the area model, draw the quantities of brownies that he will take to the party. Be sure to include the fact that each pan is divided into 12 pieces.

SUMMARY (What I learned in this section)

FRACTIONS 4

Name:_________________________ **Date:**_______ **Period:**______

SECTION 4.4 Addition and Subtraction of Fractions

VOCABULARY

DEFINITION	EXAMPLE
Least Common Denominator:	

Big Idea: How do we add and subtract fractions?

EXPLORATION 1: ADDING AND SUBTRACTING FRACTIONS

Adding 1 foot to 2 feet equals 3 feet. Combining 1 apple with 2 apples gives 3 apples. In each case, both numbers and units are important. Given these two examples, it seems reasonable to say that the sum of 1 fifth and 2 fifths is 3 fifths. More precisely, in Chapter 2, the linear skip counting model demonstrated that $\frac{3}{5}$ is $\frac{1}{5} + \frac{1}{5} + \frac{1}{5}$. Using skip counting, it is easy to see that

$$\frac{2}{5} + \frac{1}{5} = \left(\frac{1}{5} + \frac{1}{5}\right) + \frac{1}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$$

Let's try some more. In the table below, write out the addition problem using unit fractions. Record the sum in simplest form as a proper fraction or mixed number, if possible. The first one is done for you.

Addition Problem	Unit Fractions	Sum in Simplest Form or Mixed Number
$\frac{2}{3} + \frac{2}{3}$	$(\frac{1}{3} + \frac{1}{3}) + (\frac{1}{3} + \frac{1}{3}) = \frac{4}{3}$	$1\frac{1}{3}$
$\frac{3}{5} + \frac{1}{5}$		
$\frac{5}{8} + \frac{7}{8}$		
$\frac{3}{4} + \frac{3}{4}$		

How is the sum $\frac{1}{5}+\frac{2}{5}$ computed using the area model? Use a candy bar model. Betsy had $\frac{1}{5}$ of a candy bar, and her friend had $\frac{2}{5}$ of a candy bar like Betsy's.

Together, they have $\frac{3}{5}$ of a candy bar. Express this as $\frac{1}{5}+\frac{2}{5}=\frac{3}{5}$.

Write rules to generalize the previous discussion of adding fractions.

____

____

____

EXAMPLE 1

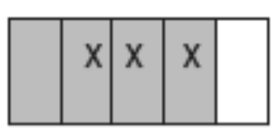
Find the sum and put in simplest form or, if possible, as a mixed fraction. Draw a model to illustrate each.

a. $\frac{4}{12}+\frac{5}{12}$

b. $\frac{3}{5}+\frac{3}{5}$

c. $\frac{2}{3}+\frac{2}{3}+\frac{2}{3}$

The same principle applies when subtracting fractions. $\frac{4}{5}-\frac{3}{5}=\frac{1}{5}$



Find the difference and put in simplest form. Use models to illustrate each. Remember the order of operations.

a. $\frac{5}{6} - \frac{2}{6}$

b. $\frac{7}{10} - \frac{2}{10}$

c. $\frac{5}{8} - \frac{3}{8} - \frac{2}{8}$

Compute $\frac{7}{9} - \frac{4}{3}$ and explain how to obtain the answer.

EXAMPLE 1

The sum of two fractions with like denominators, $\frac{a}{d}$ and $\frac{b}{d}$, is given by

$$\frac{a}{d} + \frac{b}{d} = \frac{a+b}{d}$$

Write an example to prove the above statement true.

EXAMPLE 2

Describe how to subtract fractions with like denominators. What is the difference, $\frac{m}{n} - \frac{k}{n}$?

How does your method compare to the addition rule above?

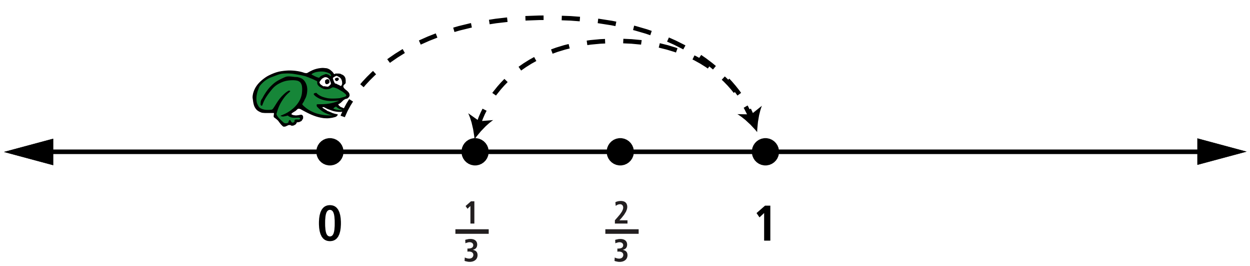
EXAMPLE 3

If you eat $\frac{2}{3}$ of a candy bar, how much of the candy bar is left? How can you use subtraction of fractions to answer this question?

Use the rectangle below to model the candy bar problem.

What happens when you subtract $\frac{2}{3}$ of the candy bar? Cross out the portions that are subtracted to show the difference.

Another way to think of this subtraction is using the frog model. Like the model for subtracting integers, the frog hops 1 unit to the right and then hops backwards a distance of $\frac{2}{3}$ to land on the number $\frac{1}{3}$. This model represents $1 - \frac{2}{3}$.



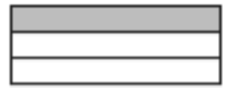
EXPLORATION 2: UNLIKE FRACTIONS

Let's explore how to use the ideas just learned to compute the sum of two fractions when the denominators are not the same.

Use the area model to compute $\frac{1}{2} + \frac{1}{3}$.

Begin by looking at a visual representation.

$\frac{1}{2} =$ 

$\frac{1}{3} =$ 

Is it possible to combine the shaded amounts? In Section 4.2, you discovered that in comparing the fractions $\frac{1}{2}$ and $\frac{1}{3}$, it was helpful to find equivalent fractions for both $\frac{1}{2}$ and $\frac{1}{3}$ to determine which is greater. Modify the picture above to display equivalent divisions of the whole.

Remember, fractional parts must be equal. A third is not the same amount as a half.

To do this, divide the first model horizontally to represent $\frac{1}{2}$ as 3 parts out of 6 parts. Then, divide the second model vertically to represent $\frac{1}{3}$ as 2 parts out of 6 parts. It is easy to see from the model that $\frac{1}{2} = \frac{3}{6}$ and $\frac{1}{3} = \frac{2}{6}$.

Using the rule for adding fractions with like denominators, the sum is:

$$\begin{array}{r} \frac{1}{2} = \frac{3}{6} \\ +\frac{1}{3} = \frac{2}{6} \\ \hline \frac{5}{6} \end{array}$$

In order to add the fractions, find common-sized pieces so that the two fractions can be written with the same, or common denominator.

The most important thing to remember when adding fractions is to ensure that you have a common denominator.

EXAMPLE 4

Compute the sum $\frac{1}{3}+\frac{1}{4}$ by first using the area model and then the equivalent fractions property to convert the fractions into equivalent fractions with like denominators.

EXAMPLE 5

Find the pattern to add the fractions $\frac{1}{a}$ and $\frac{1}{b}$ and show the process.

EXPLORATION 3: FINDING COMMON DENOMINATORS

Find three common denominators for the fractions $\frac{1}{6}$ and $\frac{1}{4}$. Write each fraction in equivalent forms using the three denominators. What do you notice about these common denominators? Which denominator would be the best choice for computing the sum $\frac{1}{6}+\frac{1}{4}$? Why?

What three common denominators did you choose? ________, ________, ________

Write 3 equivalent fractions for the unit fractions $\frac{1}{6}$ and $\frac{1}{4}$.

$\frac{1}{6}$ = ________, ________, ________

$\frac{1}{4}$ = ________, ________, ________

What do you notice about these common denominators?

Which denominator would be the best choice for computing the sum $\frac{1}{6}$ and $\frac{1}{4}$? ________

Explain your reasoning.

EXAMPLE 6

Let's review finding least common multiples. Find the least common multiple of:

a. 3 and 6 ___	c. 9 and 15 ___	e. 2 and 8 ___
b. 12 and 48 ___	d. 5 and 7 ___	f. 9, 6 and 12 ___

When we add and subtract fractions, having a common denominator is very useful. In order to add $\frac{1}{3}+\frac{1}{6}$, use the equivalent fraction, $\frac{2}{6}$ for $\frac{1}{3}$. The restatement of the problem $\frac{1}{3}+\frac{1}{6}$ to $\frac{2}{6}+\frac{1}{6}$ makes finding the sum of $\frac{3}{6}$ easier to determine.

EXAMPLE 7

For each of the following sums: (1) find a common multiple for both denominators, (2) use it to find equivalent fractions for each fraction, (3) compute their sum and (4) simplify your answer, if necessary.

a. $\frac{1}{9}+\frac{1}{12}$ b. $\frac{3}{8}+\frac{5}{12}$ c. $\frac{7}{12}+\frac{5}{18}$

Hint: It may be easier to rewrite the problems vertically, giving space to write the equivalent fractions with common denominators beside the original fractions. Use the boxes below to show your work. Write all answers in simplest form.

a.	b.	c.

In adding or subtracting fractions, the LCM of the denominators produces the least common denominator or LCD. Using the LCD has the advantage of working with smaller numbers.

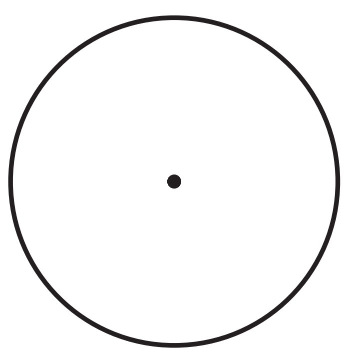
EXPLORATION 4: MODELING FRACTION ADDITION

Use the number line below to draw your model for question 1.

1. Draw a linear model to show the sum of $\frac{1}{2}$ and $\frac{1}{8}$. Explain what denominator you used and why.

2. Use an area model, for example, a rectangle, to show the sum of $\frac{1}{2}$ and $\frac{1}{8}$.

3. Use another area model, for example a circle, to show the sum of $\frac{1}{2}$ and $\frac{1}{8}$.



EXPLORATION 5

Directions: Look at the diagrams below and see if you can figure out the fractional patterns to answer each question.

1. If  = 1, then  = ______________.

2. If  = 1, then  = ______________.

3. If  = 1, then  = ______________.

4. If  = 1, then  = __________________.

5. If  +  = 1, then  = ___________.

6. If  +  = 1, what is  + ? ____________

7. If  +  = 1, what is  + ? ______________

8. If  +  = 1, what is ? ___________________

9. If  -  = 1, what is  + ? _________

PROBLEMS:

1. Raul finished $\frac{3}{5}$ of his homework before dinner. What part must he finish after dinner?

____

2. Add or subtract the following fractions:

 a. $\frac{5}{a}+\frac{7}{a}=$ ____________

 b. $\frac{7}{r}-\frac{5}{r}=$ ____________

 c. $\frac{6}{x}+\frac{5}{y}=$ ____________

3. To make punch for his party, Noah must use $\frac{4}{5}$ liter of lemon-lime soda and $\frac{5}{8}$ liter of pineapple juice. How many liters of punch will Noah make?

4. Emily made a giant sheet cake to share with friends. She gave $\frac{1}{4}$ to Kayla, $\frac{1}{3}$ to Andy, and $\frac{1}{6}$ to Victoria.

 How much cake did she give away? ____________________

 How much cake is left? ____________________________________

5. Compute and simplify. Express as a mixed number, if needed.

a. $\frac{1}{4}+\frac{3}{5}+\frac{1}{2}=$ ______

b. $\frac{1}{3}+\frac{5}{6}+\frac{7}{9}=$ ______

c. $\frac{1}{8}+\frac{1}{3}+\frac{5}{12}=$ ______

6. Tell which is greater. Use >, < or =.

a. $\frac{3}{5}+\frac{2}{3}$ ☐ $1-\frac{1}{8}$

b. $\frac{1}{3}+\frac{2}{5}$ ☐ $\frac{7}{9}-\frac{1}{3}$

c. $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$ ☐ $2-\frac{3}{5}-\frac{2}{5}$

d. $\frac{2}{5}+\frac{1}{5}$ ☐ $\frac{5}{8}-\frac{1}{4}$

SUMMARY (What I learned in this section)

FRACTIONS 4

Name:______________________________ **Date:**________ **Period:**_______

SECTION 4.5 Common Denominators and Mixed Fractions

Big Idea: How do you add and subtract mixed numbers?

EXPLORATION 1: MIXED NUMBERS

Silvia is baking six sheet cakes for a party. The recipe she is using calls for $3\frac{1}{6}$ pounds of refined sugar and $5\frac{1}{4}$ pounds of unrefined sugar. First use the linear model to estimate the total pounds of sugar Silvia needs. Then compute exactly how many pounds of sugar Silvia needs. Explain your process for both the estimation and the exact calculation. Can you use the same process to add other mixed numbers?

Use the number line below to create your linear model.

How much sugar do you estimate Silvia will need? ______________

Next, compute exactly how many pounds of sugar Silvia needs.

$$\begin{array}{r} 3\frac{1}{6} \\ +\ 5\frac{1}{4} \\ \hline \end{array}$$

Explain the process you used for the estimation and the calculation.

____

____

EXAMPLE 1

The following recipe yields about 6 dozen Chocolate Chip cookies.

CHOCOLATE CHIP COOKIES

$2\frac{1}{4}$ cups flour

$\frac{3}{4}$ cups sugar

$\frac{3}{4}$ cups brown sugar

12 oz. chocolate chips

1 tsp. baking soda

$\frac{1}{8}$ tsp. salt

$1\frac{1}{2}$ tsp. vanilla

How would you adjust the recipe to make 12 dozen cookies? How would you adjust the recipe to make only 3 dozen cookies? Use the table below to organize your calculations.

6 Dozen	12 Dozen	3 Dozen

EXPLORATION 2

Compute the sum $6\frac{3}{5}+3\frac{5}{7}$ using three different methods.

1. Improper Fractions:

One approach is to treat this as an ordinary fraction addition problem by converting from mixed to improper fractions and back again. First, convert the mixed fractions to improper fractions:

The whole number 6 is the same thing as 30-fifths. Therefore, $6 + \frac{3}{5} = \frac{33}{5}$. Likewise, $3\frac{5}{7} = \frac{26}{7}$. Now we just need to add the two fractions. Don't forget to rewrite the fractions with common denominators.

$$\frac{33}{5} = \_\_\_\_\_\_\_\_$$

$$+ \ \frac{26}{7} = \_\_\_\_\_\_\_\_$$

Because you are adding two improper fractions, your answer will also be an improper fraction. Convert your answer to a mixed number.

2. Combining Like Parts:

The improper fractions approach can be cumbersome because it involves working with relatively large numbers. Another approach is to consider each mixed fraction as the sum of an integer and a proper fraction and regroup the whole parts together and the proper fractions together:

$$6\frac{3}{5} + 3\frac{5}{7} = \left(6 + \frac{3}{5}\right) + \left(3 + \frac{5}{7}\right) = (6+3) + \left(\frac{3}{5} + \frac{5}{7}\right)$$

This leads to the sum of proper fractions:

$$\begin{aligned}\frac{3}{5}+\frac{5}{7} &= \frac{3\cdot 7}{5\cdot 7}+\frac{5\cdot 5}{7\cdot 5}\\ &= \frac{21}{35}+\frac{25}{35}\\ &= \frac{46}{35}\\ &= 1\frac{11}{35}\end{aligned}$$

Combining these results, the original sum is:

$$6\tfrac{3}{5} + 3\tfrac{5}{7} = \left(6 + \tfrac{3}{5}\right) + \left(3 + \tfrac{5}{7}\right)$$

$$= (6 + 3) + \left(1 + \tfrac{11}{35}\right)$$

$$= 10 + \tfrac{11}{35}$$

$$= 10\tfrac{11}{35}$$

As you can see, in computing the sum of mixed fractions, it is often easier to separate the mixed fractions as whole parts and fractional parts, add each group and then combine these two partial sums.

3. Vertical Addition:

There is another way to organize and write this same process vertically:

$$\begin{array}{r} 6\tfrac{3}{5} \\ +\,3\tfrac{5}{7} \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{rr} 6 & \tfrac{3}{5} \\ +\,3 & +\,\tfrac{5}{7} \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{rr} 6 & \tfrac{21}{35} \\ +\,3 & +\,\tfrac{25}{35} \\ \hline 9 & \tfrac{46}{35} \end{array} \quad \begin{array}{l} = 9 + \left(1 + \tfrac{11}{35}\right) \\ = 10 + \tfrac{11}{35} \\ = 10\tfrac{11}{35} \end{array}$$

How would finding the difference between two mixed fractions be different?

EXAMPLE 2

Compute the following differences:

a. $8\tfrac{4}{5} - 5\tfrac{3}{10}$	b. $6\tfrac{3}{5} - 3\tfrac{5}{7}$	c. $4 - 2\tfrac{3}{5}$

Remember:

- You must have like denominators to add or subtract fractions.
- Writing the problems vertically may help with organization.
- You may use any of the three methods you choose.

EXAMPLE 3

Let's try a few more addition and subtraction problems.

a. $73\frac{8}{9}+28\frac{1}{3}$ b. $29-8\frac{19}{40}$ c. $2\frac{1}{5}-1\frac{2}{3}$

Use the boxes below to show your work.

a.	b.	c.

PROBLEMS

1. Compute the following sums of mixed fractions using either the horizontal or vertical method. Show all the steps in the process. Simplify your answers if needed.

   a. $2\frac{2}{5}+4\frac{1}{5}=$ b. $4\frac{3}{8}+\frac{5}{12}=$ c. $5\frac{3}{4}+2\frac{2}{6}=$

   Use the boxes below to show your work.

a.	b.	c.

2. Compute the following differences of mixed fractions using either the horizontal or vertical method. Show all the steps in the process. Simplify your answers if needed.

a. $1\frac{3}{14} - 1\frac{1}{7} =$ b. $7\frac{3}{4} - 5\frac{2}{6} =$ c. $9\frac{3}{8} - 4\frac{5}{12} =$

Use the boxes below to show your work.

a.	b.	c.

3. Kyra has $2\frac{3}{4}$ liters of soda. Jared has $1\frac{5}{8}$ liter of sports drink. How much liquid do they have all together?

How much more drink does Kyra have than Jared?

4. On Monday it rained $\frac{3}{4}$ of an inch in Austin, while Dallas received $\frac{5}{6}$ of an inch.

Which city received more rain? ______________________________

How much more rain? ____________________________________

Together, how much rain fell in the cities? ____________________

5. Nicole babysat her younger cousins for $2\frac{1}{4}$ hours on Monday, for $1\frac{1}{2}$ hours on Wednesday and $3\frac{3}{5}$ hours on Friday. How much total time did she spend babysitting her cousins?

____

6. Sophia's puppy, Boo boo, has 1 cup of food in her dish. If she eats $\frac{4}{7}$ of the food in her dish, how much remains?

____

7. Cameron has $6\frac{1}{3}$ yards of rope. He cuts off $4\frac{3}{8}$ yards of the rope to make a rope swing at the river. How many yards of rope does Cameron have left?

____

SUMMARY (What I learned in this section)

FRACTIONS 4

Name:__________________________ **Date:**_______ **Period:**______

CHAPTER 4: SPIRAL REVIEW

1. Evaluate the following expression using the Order of Operations:

$$10 - 2 \cdot 4 \div 2^3 \cdot 5 + 3$$

____

____

____

____

2. Write an expression for each statement below, and then evaluate:
   a. 3 more than 8 times 5 ____
   b. 5 less than 30 divided by 2 ____________________________________
   c. 6 times the quotient of 8 and 4 _________________________________
   d. 14 less than double 100 _______________________________________

3. Karmen gets regular shipments from Parcel Express. Every 12 days she gets a shipment of her favorite chocolate. Every 15 days she receives a new stock of her kitten's gourmet cat food. And every 6 days she gets a fresh supply of pineapples. If she just received all three shipments today, when is the next day she will again receive all three on the same day?

   ____

4. Convert each of the mixed numbers to an improper fraction:

Mixed Number	Improper Fraction	Mixed Number	Improper Fraction	Mixed Number	Improper Fraction
$\frac{5}{12}+\frac{7}{12}+1\frac{1}{2}$ =		$30\frac{2}{5}$ =		$7\frac{3}{5}$ =	
$6\frac{3}{8}$ =		$9\frac{5}{9}$ =		$8\frac{1}{2}$ =	
$11\frac{5}{8}$ =		$10\frac{7}{9}$ =		$20\frac{6}{7}$ =	

5. Finish the statement: In order to add or subtract fractions you must ______________________

____

______________________.

6. Add or Subtract the following whole numbers, fractions, or mixed numbers. Be sure to show your work.

$\frac{6}{7}+\frac{5}{9}=$	$1\frac{5}{9}-\frac{3}{8}=$	$3\frac{1}{2}+2\frac{8}{13}=$
$\frac{5}{11}-\frac{1}{3}=$	$7-\frac{11}{13}=$	$\frac{5}{16}+4=$

7. Rewrite the fractions with a common denominator in order to compare. Write the fractions in original form from least to greatest.

   a. $\frac{5}{12}, \frac{7}{10}, \frac{4}{5}$ ______________________________

   b. $\frac{3}{4}, \frac{5}{8}, \frac{4}{9}$ ______________________________

   c. $\frac{3}{5}, \frac{1}{8}, \frac{2}{3}$ ______________________________

8. Randyn jogged $5\frac{1}{4}$ miles, Shane jogged $\frac{14}{3}$ miles, and Amy jogged $3\frac{15}{12}$ miles.

   Who jogged the farthest? Explain your answer.

   ____

9. Write 612 as a product of its prime factors. ______________________________

10. Show how you can use unit rates to add $\frac{5}{12}+\frac{7}{12}$:

____

____

____

____

DECIMAL AND PERCENT REPRESENTATIONS 5

Name:__________________________________ **Date:**_______ **Period:**______

SECTION 5.1 Constructing Decimals

Big Idea: How do you compare, order, and operate with decimal numbers?

EXPLORATION 1: DECIMAL PLACE VALUE

Throughout elementary school, we mainly work with whole numbers. Yet, money introduces us to the concept of amounts less than 1. While dollar bills represent whole values, the change we get back from a purchase represents parts of a whole. For example, it takes ten dimes to make a dollar. Therefore, one dime is one-tenth of a dollar.

Let's look at a place value chart:

PLACE VALUE CHART

THOUSANDS	HUNDREDS	TENS	ONES	•	TENTHS	HUNDREDTHS	THOUSANDTHS

What similarities do you notice on the chart?

What differences do you notice?

Think about a nickel. You write its value as 0.05. Because the last digit ends in the hundredths place, you read this as "five-hundredths".

1. Write the following in decimal form. Refer to the place value chart as needed.
   a. Thirty-six hundredths ______________
   b. Five thousandths ______________
   c. Six tenths ______________
   d. Four hundredths ______________
   e. Twenty-nine thousandths ______________

2. Write the following decimals in word form.

   a. 0.07

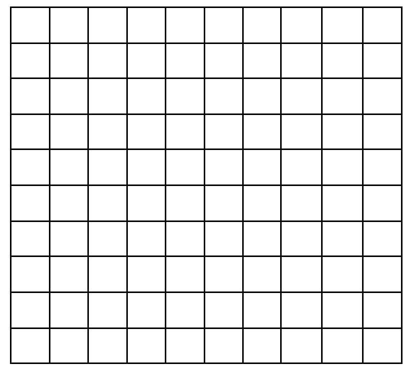
   ____

   b. 0.001

   ____

Sometimes, whole numbers are part of the number. Consider 1.37. This is read, "one AND thirty-seven hundredths". When reading a decimal number, the word "AND" represents the decimal.

EXPLORATION 2: MODELING DECIMALS



We can use decimal grids to model decimals.

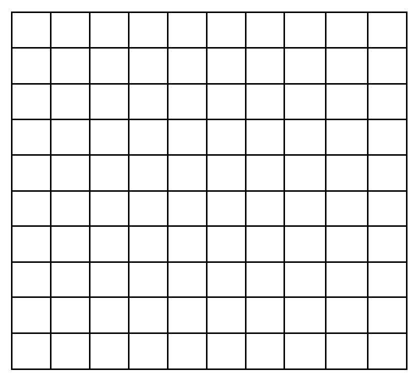
How many columns are in the grid? ______ How many small squares are in a column? ________ How many small squares are in the grid altogether? _____

Shade in one of the columns.

What fraction of the grid is now shaded? ______ Be sure to write your fraction in simplest form. Convert your fraction to a decimal: _______

Next, shade 7 more small squares. You should have one column and 7 small squares shaded. What fraction of the grid is shaded altogether? ________

Convert your fraction to a decimal: _______



Let's try modeling on the hundredths grid.

First, shade thirty-two hundredths on the grid. Next, use a different color without overlapping the first shading and color in four-tenths of the grid.

Write the number sentence for finding the sum of these two amounts:

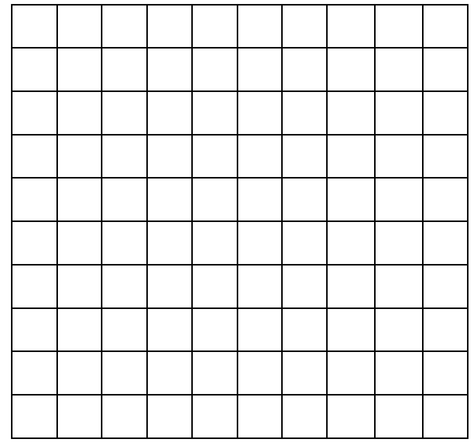
____________________________.

You can also use the grid to show subtraction. Consider the problem "Five tenths take away twenty-one hundredths". You will first shade in the five-tenths.

Next, to model subtraction, you will draw an "*x*" over the 21-hundredths.

Now write the number sentence, or equation, that you modeled above:

______________________________________.



EXPLORATION 3: LOCATING DECIMAL NUMBERS ON A NUMBER LINE

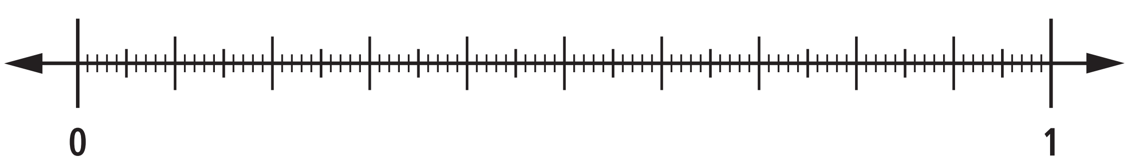
If we think of the number 1 on the number line as $1.00, where would we locate half a dollar or $0.50? Because there are 10 dimes in a dollar, where would $0.10 be located on the number line? $0.20? $0.30? Can you locate $0.01 or more simply 0.01 on the number line, knowing that there are 10 pennies in a dime?

We know when we write the number 0.30 that there is another way that this decimal can be written. Thirty hundredths can be written as 0.3. How could you show the two numbers 0.3 and 0.30 are really equivalent to each other on the number line?

Use a number line like the one below to find the locations of the following decimal numbers. Notice that 0 and 1 are labeled on the number line.

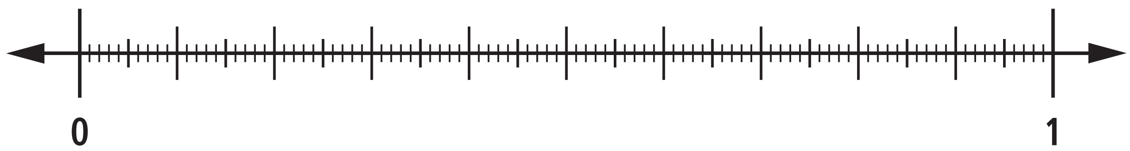
a. 0.4 c. 0.68

b. 0.27 d. 0.7



For each pair of numbers, determine whether the numbers are equal or not and place the appropriate sign, >, <, =, between the numbers. Justify your answer using the number line.

a. 0.68 ☐ 0.7 b. 0.34 ☐ 0.339 c. 0.268 ☐ 0.271

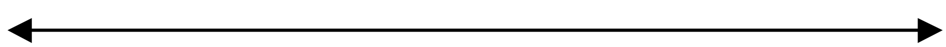


Remember what you have learned about decimals. You know that the number line is one tool that can help you locate decimals, order them, and compare them. The place value chart is another tool that can help you do this. Let's take a look at the next example to understand how the place value chart may be useful.

EXAMPLE 1

In a science project, Jeremy measured the distance two cars traveled. Car A traveled 1.38 meters and Car B traveled 1.4 meters. Which car traveled further? ______________________________

Mark a segment of the number line below to illustrate where Car A and Car B stopped.



EXPLORATION 4: ROUNDING DECIMAL NUMBERS

Like whole numbers, decimals can also be rounded to a specific place value. Consider the number 18.625. If you were asked to round this to the nearest tenth, you begin by underlining the 6 in the tenths place, $18.\underline{6}25$. Then, you look to the right of the underlined digit to determine if you should round up or down. Rounding up allows you to increase the underlined digit by 1 while rounding down means the underlined digit remains the same. In our example, the 2 means that the underlined 6 will round down or stay the same: 18.625 will round to 18.6. An equivalent answer is 18.600 but the ending zeros are generally dropped when a decimal number ends in zeros.

To round decimals:

1. Find the place value you want to round to. Call this the specified digit (The underlined place in our previous example above). Look at the digit to the right of it.

2. If the digit to the right is less than 5, do not change the specified digit and drop all digits to the right of it.

3. If the digit to the right is 5 or greater, add one to the specified digit and drop all digits to the right of it.

EXAMPLE 2

Following the instructions just given, round 21.093 to the nearest hundredth.

21.093 rounds to ________________.

Round the following numbers to the nearest tenth.

a. 16.709 ____________ b. 10.995 ____________ c. 0.471 ____________

d. 1.92587 ____________ e. 102.08 ____________ f. 72.1 ____________

EXPLORATION 5: ORDERING DECIMAL NUMBERS

When alphabetizing two or more words, you ignore beginning letters that are alike until the first different letter to determine which word comes before another. For example, concentrate, concert, and cone are in alphabetical order. Can you see why? Similarly, when ordering numbers, compare the digits starting with the largest place value. Ignore the digits that are alike until the first place value that shows a difference. This process is often easiest to see if you make a vertical column of the numbers you are ordering.

Write the following numbers in order from least to greatest: 3.065, 3.6, 3.56, 3.605, 3.65. Use the box given to align the decimals of each number.

	•			
	•			
	•			
	•			
	•			

EXAMPLE 3

Try a few more on your own. Order the next sets of numbers from greatest to least.

a. 1.1, 1.095, 1.9, 1.59 ____

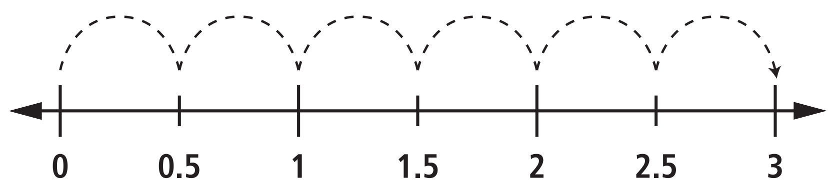
b. 5.1823, 5.1, 5.18023, 5.108 ______________________________________

c. 6.08, 6.008, 6.8, 6.088 ____

EXPLORATION 6: DIVIDING WHEN THE DIVISOR IS GREATER THAN THE DIVIDEND

Sarah spent \$3 on 6 chocolate bars. How much did each candy bar cost?

This is a typical division problem where the cost of each candy bar is $3 \div 6$. You might expect trouble because the divisor is greater than the dividend. Using the linear skip counting model, how long does each skip need to be to travel a distance of 3 units, or 3 dollars in this case, in 6 skips?



You can see that each jump is \$0.50 or half a dollar. This represents the fact that each candy bar costs \$0.50. If necessary, verify this using the calculator by computing $3 \div 6$ or adding six skips 0.50 long. The long division method gives us the same result, because there is a decimal point before the 5. You might write the problem like step 1 on the next page. The divisor is greater than the dividend, so modify the long division process by placing a decimal point. Add the two zeros in the dividend because we are working with money and we know that \$3 = \$3.00 where \$0.00 represents no cents. Where does the decimal place appear in the quotient? Why does this make sense?

EXAMPLE 4

Look at the steps below used to compute the amount without the use of a number line.

Step 1: $6\overline{)3}$

Step 2:

$$\begin{array}{r} 0.50 \\ 6\overline{)\,3.00} \\ -3.00 \\ \hline 0.00 \end{array}$$

Where does the decimal place appear in the quotient? ________________________________

Try using long division to solve the following:

a. $1 shared among five friends.	b. 15 divided by 20

Compute the following division problems by using an abbreviated number line from 0 to 2, like the one below, and finding the quotient using the skip-counting method. Then use the scaffolding method to verify your answer. Make sure the decimal point in the quotient makes sense in the context of the problem. Then use the calculator to confirm your work, if necessary.

a. $1 ÷ 5

b. $1.60 ÷ 8

c. $1.20 ÷ 4

You will be seeing more decimal division in Section 5.2!

PROBLEMS

1. For each pair of numbers, determine which is greater. Justify your answer using a number line.
   a. 0.52 and 0.09 ____________

   b. 0.82 and 0.819 ____________

   c. 0.268 and 0.259 ____________

   d. 1.12 and 1.02 ____________

   e. 3.45 and 3.045 ____________

   f. 2.133 and 2.10 ____________

2. Round the following number 127.398359 to the specified place value:

   a) nearest one's ______________

   b) nearest tenths ______________

   c) nearest hundredths ______________

   d) nearest thousandths ______________

3. The cost of a 50-inch flat screen television on sale is $719.29.

   a) Round the cost to the nearest dollar. _________

   b) Round the cost to the nearest hundred dollars. __________

4. Mr. Garza has some money in his pocket that he intends to divide equally among his four nephews. Use the area model and the scaffolding model to compute how much each nephew receives if he has
   a. $26 in his pocket ______________

   b. $27.40 in his pocket ______________

5. Complete the table below.

Decimals	Words
0.017	
	Five and one hundred six thousandths
0.9	
	One hundred two and six thousandths

6. Order the decimals from least to greatest by vertically stacking the numbers.

a. 8.75, 8.705, 8.075, 8.7	b. 16.97, 17, 16.909, 16.979	c. 85.7, 84.9, 85.78, 84.987

7. Nancy has $10 to spend at the movies. If tickets to the matinee cost $2.50, how many tickets can she buy?____

SUMMARY (What I learned in this section)

DECIMAL AND PERCENT REPRESENTATIONS 5

Name:________________________________ Date:_______ Period:______

SECTION 5.2 Operating with Decimals

Big Idea: How do we add, subtract, multiply, and divide decimal numbers?

EXPLORATION 1: ADDITION OF DECIMALS

We'll start this section dealing once again with money. Consider subtracting 3 cents from $35.50.

When you subtract 3¢ from $35.50, you are really subtracting $0.03 from $35.50 to get $35.47. As with addition, it is important to keep in mind the place value and subtract the hundredths from the hundredths, the tenths from the tenths, and so forth. You might have heard the phrase "line up the decimals." This vertical, stacking method assures that the place values also line up to do the calculation.

```
   29.90
    3.49
    1.09
  + 0.99
________
   35.47
```

EXAMPLE 1

Betty is about to take a trip. She fills her car with gas for $29.90 and buys a map for $3.49, a drink for $1.09 and a pack of gum for $0.99. Estimate the cost of her purchase before taxes. Is $40.00 enough to pay for the purchase, excluding tax?

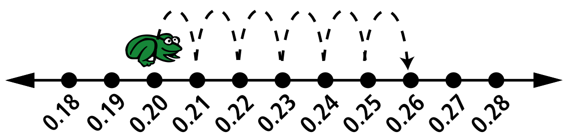
Now calculate the exact cost of her purchase before tax. Remember to align the decimals vertically in order to keep the digits in the correct place value.

Betty's actual cost for her purchase will be ____________________.

EXPLORATION 2: LINEAR MODEL

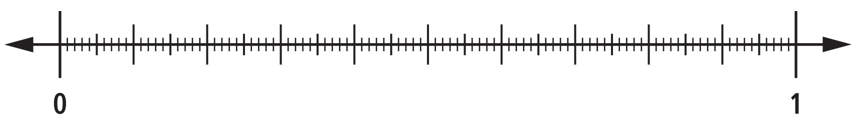
You can also use the number line to add decimal numbers.

For example, 0.2 + 0.06 = 0.20 + 0.06.



Explain how the number line can help to estimate a sum before you calculate the actual total.

Compute the sums in parts a & b below using the number line. Compute c & d using the stacking method.



a. 0.2 + 0.05 = ____________	b. 0.24 + 0.03 =___________
c. 13.7 + 7 = ________	d. 12.5 + 3.075 = _________

How do you use the number line to subtract decimals? Compute the following differences using the number line, and then subtract using the traditional stacking method.

a. 0.63 – 0. 07 _______________

b. 0.2 – 0.06 _______________

EXPLORATION 3: MULTIPLYING DECIMAL NUMBERS

Marie discovers a sale at her favorite store. A shirt she really likes is on sale for $11.59, so Marie decides to buy one shirt in each of her favorite colors: fuchsia, purple, and blue. How much did the 3 shirts cost before tax?

First, let's estimate to see about how much Marie paid. Is $11.59 closer to $11 or $12?

Use your estimate ☐ · 3 = _________. This amount is your estimate for the cost of the 3 shirts.

Now, let's insert the real price: 11.59 · 3

Step 1: multiply the numbers as if they are integers and ignore the decimal points for now.	1159 x 3 3477
Step 2: Count the number of decimal places in each of your factors and add the decimal place of the factors.	11.59 (2 decimal places) x 3 (0 decimal places) (2 decimal places)
Step 3: Place the decimal point in your answer found in Step 1 by: a. starting at the right. b. moving left the number of places equal to the sum of the number of decimal places of the factors as found in Step 2. c. placing the decimal point in that location.	11.59 x 3 34.77 (2 decimal places)
Step 4: Check to see if your answer sounds reasonable and close to your estimate above.	

EXAMPLE 2

Now, let's try multiplying two decimal numbers to see what happens.

Compute $15.22 \cdot 2.3$

First, we estimate $15 \cdot 2 =$ ___

Now, let's insert the real numbers:

Step 1: It is not necessary to align the decimals. In fact, we can ignore the decimals for now and multiply as usual.	1522 x 23
Step 2: Now we deal with the decimal. Count the number of decimal places in the factors. **Step 3:** Start at the right of your product in Step 1. Move to the left the sum of the number of places in your factors and place the decimal point at that location.	15.22 (_____ decimal places) x 2.3 (_____ decimal places) (_____ decimal places)
Step 4: Check to see if your answer sounds reasonable and close to your estimate above.	

Notice that we do NOT have to line up decimals to multiply.

Try some on your own

a. 8.33 · 5 =	b. 12.3 · 2.4 =	c. 20 · 4.9 =
d. 9.075 · 0.2 =	e. 7.65 · 1.2 =	f. 11.1 · 4.8 =
g. 14.92 · 2.31 =	h. 6.32 · 4.18 =	i. 7.42 · 8.6 =
i. 67.22 · 3.456 =	j. 5.116 · 54.3	k. 9.644 · 2.2

EXPLORATION 4: DIVIDING DECIMAL NUMBERS

Anastasia bought 15 CDs for a total of $185.85. What is the average cost of each CD?

We recognize this is a division problem because they tell us the total and ask for the average cost of each.

Use the space below to solve $185.85 ÷ 15.

$15\overline{)185.85}$

The average cost of 1 CD is ____________.

EXPLORATION 5: DIVIDING WHEN THE DIVISOR IS GREATER THAN THE DIVIDEND

Vance found that he had $32.75 in his bank. He wants to take it out all in quarters. How many quarters are in $32.75?

To solve this, we have to find out how many times 0.25 will go into $32.75. We set our problem up like this:

"32.75 divided by 0.25"

$$0.25\overline{)32.75}$$

Before we can divide, however, we notice there is a decimal in the divisor. We want to convert 0.25 to a whole number in order to continue. If we multiply 0.25 times 100, it will move the decimal. Try it:

100	0 places
x 0.25	2 places
	2 places

Now we have a whole number in the divisor. To keep our equation balanced, we must perform the same operation on the dividend.

$$\$32.75 \cdot 100 = \$3,275$$

Let's see what we have done to our original problem:

$0.25 \cdot 100\overline{)32.75 \cdot 100}$ becomes $25\overline{)3275}$

We now have a whole number in the divisor and we can perform our operation. We find that there are

_____ quarters in $32.75.

Try some more on your own.

a. 20 ÷ 2.5	b. 40.25 ÷ 0.5	c. 0.36 ÷ .08
d. 0.72 ÷ 0.08	e. 124 ÷ 0.12	f. 52 ÷ 1.25
g. 210 ÷ 4.2	h. 180 ÷ 2.4	i. 180 ÷ 0.24
j. 450 ÷ 1.8	k. 197.03 ÷ 1.75	l. 0.324 ÷ 3.6

PROBLEMS

1. What must you always do before adding or subtracting decimal numbers?

____

2. Compute the following:

a. 0.67 + 0.54	b. 0.93 + 9.12	c. 27.13 + 1.9
d. 0.75 – 0.24	e. 7.602 – 3.12	f. 89.42 – 4.63
g. 0.665 + 3.45 + 2.40	h. 56.43 + 24.110 + 1.24	i. 67.34 + 2.359 + 0.911

3. Determine which of the following pairs of numbers is closer together. Explain your answer.
    a. 0.4 and 0.5 OR 0.41 and 0.45
    b. 0.79 and 0.81 OR 0.792 and 0.801

4. Alexandria has $423.79 in her savings account. She wants to buy a shirt for $17.99 and jeans on sale for $68 including tax. If she buys these clothes, how much will be left in her savings account?

5. When you multiply decimal numbers, is it necessary to line up the decimals? _________. Explain.

6. Find the product of 1.5*y*, if *y* is equal to the values given below.

a. $y = 0.3$	b. $y = 10$	c. $y = 0.09$
d. $y = 12.7$	e. $y = 15.114$	f. $y = 0.007$

7. Gil has 153.75 fluid ounces of hydrogen peroxide to pour evenly into 15 flasks. How many ounces will he pour into each flask?

8. Parker has 29.75 fl. oz. of fluid to pour into flasks. Each flask must have 1.75 fl. oz. How many flasks can he fill?

SUMMARY (What I learned in this section)

DECIMAL AND PERCENT REPRESENTATIONS 5

Name:_______________________________ **Date:**_______ **Period:**______

SECTION 5.3 Numbers as Decimals and Fractions

VOCABULARY

DEFINITION	EXAMPLE
Equivalent:	
Fraction:	

Big Idea: How can we represent the same quantity in decimal and fraction forms?

In the past, you have probably referred to one-half of a dollar as \$0.50 or 50 cents. One half is a fraction that is equal to 0.50, a decimal. We say that $\frac{1}{2}$ is **equivalent** to 0.50. In this section, we will review how a fraction can be represented as a decimal number and how some decimals can be represented as fractions.

EXPLORATION 1: DECIMAL NUMBERS AND THEIR NAMES

Let's begin this exploration by recreating the place value chart. Write the names of the place values in the chart below, as shown in the example below.

THOUSANDS			ONES	•	TENTHS		

Now, read the following words and write the decimal form in your place value chart.

a) Fifty and five thousandths

b) Two and nine hundredths

c) Four thousands six and twenty-five hundredths

d) Sixteen and sixteen thousandths

e) Three hundred twenty and seven tenths

Decimals can be read in terms of the place values that the digits occupy. For example, 0.47 is read "forty seven one hundredths." You know that the fractional representation $\frac{47}{100}$ is also read "forty seven one hundredths." You can also locate the numbers on the number line as the same point. The decimal form 0.47 and the fractional form $\frac{47}{100}$ actually represent the same value.

In converting a decimal to a fraction, we take advantage of the fact that we use the base ten system to write each decimal number. For example, the number 0.3 is called three tenths and so is equivalent to $\frac{3}{10}$. The number 0.35 is read as 35 hundredths and so is the same as $\frac{35}{100}$.

EXPLORATION 2: DECIMALS TO FRACTIONS

Does the fraction $\frac{1}{5}$ have a decimal form? If we buy 5 bananas for \$1, we know that each banana costs \$1 ÷ 5 = \$0.20 or 20 cents. In other words, each banana costs $\frac{1}{5}$ dollar because it takes 5 (\$0.20) to make a whole dollar. So 1 ÷ 5 = 0.20 or 20 hundredths. But the decimal 0.20, or twenty hundredths, has the same name as the fraction $\frac{20}{100}$. Does this mean the fraction $\frac{1}{5}$ is equal to $\frac{20}{100}$? These are equivalent fractions, so the decimal 0.20 and the fractions $\frac{1}{5}$ and $\frac{20}{100}$ all represent the same quantity and are equal.

Write the fractional form of the following decimal numbers. It may help to first say the word correctly using place value. Remember to simplify all fractions, if possible.

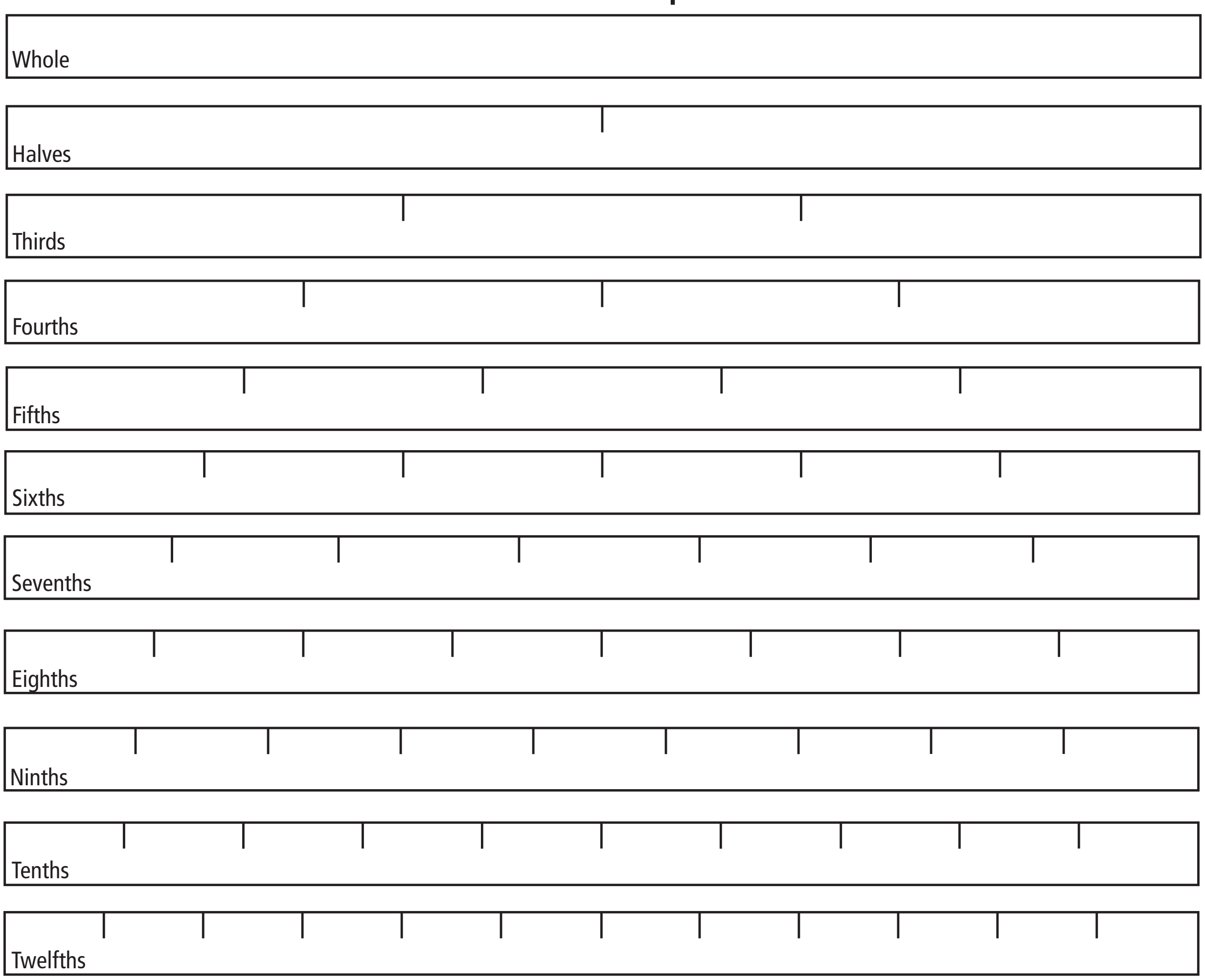
a) 0.7	b) 0.01	c) 0.216
d) 0.903	e) 5.4	f) 4.7

EXPLORATION 3: LINEAR MODEL FOR FRACTIONS

On a number line, each integer corresponds to a point. Recall that there are many other points between each pair of integers on the number line, and each of these points also corresponds to a number. We will now construct a number line from 0 to 1.

Use the number line above as your line master. The rows of the Fraction Strips Chart below represent the result of folding the whole into equal parts to represent fractions as we did in the Linear Model Activity. The computer that created it allows the folds to be very accurate. For example, using the Fraction Strips Chart, determine which fraction is greater: $\frac{2}{5}$ or $\frac{3}{7}$.

Fraction Strips Chart



EXAMPLE 1

Convert the following fractions to decimal form. You may refer to your linear model as needed.

a. $\frac{1}{4} =$

$\frac{2}{4} =$

$\frac{3}{4} =$

b. $\frac{1}{5} =$

$\frac{2}{5} =$

$\frac{3}{5} =$

$\frac{4}{5} =$

c. $\frac{1}{8} =$

$\frac{2}{8} =$

$\frac{3}{8} =$

$\frac{5}{8} =$

d. $\frac{1}{3} =$

$\frac{1}{30} =$

$\frac{1}{9} =$

$\frac{1}{90} =$

EXAMPLE 2

We see that the fraction bar $\frac{m}{n}$ is just another symbol used for division, m ÷ n. To convert a fraction to a decimal, it helps to remember that a fraction is a division problem. $\frac{1}{4}$ is the same thing as 1 ÷ 4.

Consider the next example involving money.

If you buy four apples for a dollar, how much does each apple cost? Dividing \$1 by 4 yields, \$1÷4 = \$0.25, or 0.25 dollars. We can also say each apple costs a quarter or $\frac{1}{4}$ of a dollar. So $\frac{1}{4}$ and 0.25 are equal or equivalent. Now we ask, "What decimal is equivalent to $\frac{1}{3}$?" We could also ask what is $\frac{1}{3}$ of a dollar? We can use our new rule to see that $\frac{1}{3}$ is equivalent to the quotient of 1 ÷ 3. The quotient is a repeating decimal, 0.33333... which can be written as $0.\overline{3}$. The bar over the 3 tells us that the digit 3 repeats without end. Previously, we discovered that this quotient is $1 \div 3 = 0.3333\ldots = 0.\overline{3}$. While $\frac{1}{3}$ of a dollar is $\$0.\overline{33}$, we cannot practically divide \$1 into 3 equal parts with our present set of coins, so we often approximate to $\$0.\overline{33}$.

There are other fractions that equal repeating decimals:

$2 \div 3 = 0.6666... = 0.66\overline{6} = 0.\overline{6}$

$1 \div 6 = 0.1666... = 0.166\overline{6} = 0.1\overline{6}$

PROBLEMS

1. Complete the missing parts in the table below.

Fractions	Decimals	Words
	0.5	Five tenths
	0.35	
$\frac{3}{4}$		
		Six and three one hundredths
$8\frac{3}{12}$		

2. Order the following numbers from least to greatest and plot them on the number line below: 0.25, $\frac{9}{10}$, 0.725, $\frac{3}{5}$

3. The following racers are listed beside their time. Write the names of the winners from first to last place. (Remember the fastest person has the smallest time.)

Mimi	7.324 minutes	1^{st}: ______
Stu	7.3 minutes	2^{nd}: ______
Chandra	7.1001 minutes	3^{rd}: ______
Tess	6.2 minutes	4^{th}: ______
Eric	6.11 minutes	5^{th}: ______

4. Determine whether a decimal or fraction representation is more appropriate in the following situation.

   a. Leila is baking a cake: $2\frac{1}{2}$ cups of sugar or 2.5 cups of sugar?

b. Patricia is putting gas in her car: $12\frac{3}{5}$ gallons or 12.6 gallons?

c. You are saving money: a $12.50 deposit or $$12\frac{1}{2}$ deposit?

5. Convert each decimal to an equivalent fraction or mixed number. Simplify if needed.

a. 0.8	b. 0.07	c. 4.019
d. 0.38	e. 1.55	f. 3.005
g. 0.84	h. 50.37	i. 2.12

6. Taylor measured the width of the spine of her Math Explorations textbook and found it to measure between 2.5 and 2.7 cm. Name 3 possible measurements for the width of the spine:

_______________, ________________, __________________

7. Justin's sandcastle measured $8\frac{1}{4}$ feet tall while Kliftan's measured $7\frac{3}{5}$ feet tall. What is the difference in height between these two amazing sandcastles? ______________________

SUMMARY (What I learned this section)

DECIMAL AND PERCENT REPRESENTATIONS 5

Name:_________________________________ Date:_______ Period:______

SECTION 5.4 Fractions, Decimals, and Percent

VOCABULARY

DEFINITION	EXAMPLE
Percent:	

Big Idea: How do we represent quantities in fractional, decimal, and percent form?

EXPLORATION 1: FRIENDLY FRACTIONS

You have learned that fractions such as $\frac{3}{4}$ can be written as the equivalent fraction $\frac{75}{100}$. This equivalent fraction can also be represented by the decimal 0.75. In some instances, this number can then be converted to 75 percent, 75%. The word percent means "out of a hundred" in Latin.

Decimals can be converted to fractions by reading the decimal form. For example 0.75 is read "seventy-five one hundredths" which in fractional form is 75/100. This in turn says 75 out of 100 or 75%. Notice how three different forms, the decimal, fractional, and percent are all referring to the same quantity.

Similarly, you can reverse the pattern of converting percentages to decimals by dividing the percent by 100.

For example 75% is equivalent to $75 \div 100 = \frac{75}{100} = 0.75$. Even if the percent includes a decimal part, simply divide by 100 to get its decimal equivalent. For example, 6.48% is equivalent to $6.48 \div 100 = 0.0648$.

FRACTION	DECIMAL	PERCENT
$\frac{3}{4}$	0.75	(0.75) (100)= 75%
$\frac{12}{25}$	0.48	(0.48) (100)= 48%
$\frac{7}{10}$		
$\frac{3}{15}$		
$\frac{6}{1000}$		
$\frac{9}{20}$		

While the process used in the table is useful for many fractions, it only works for "friendly fractions." A friendly fraction is one that can easily be converted to fraction form with denominators that are tenths, hundredths, and thousandths.

How do you convert a fraction like $\frac{15}{20}$ into a decimal and a percent?

Since the denominator 20 can easily convert to 100, let's write an equivalent fraction with 100 as the denominator.

Once you know the hundredths, convert to decimal form. To convert to a percent, multiply the decimal value times 100.

$\frac{15}{20} = \frac{?}{100} =$ __________ (Decimal) = ____________ (percent)

EXAMPLE 1

A class of 25 students has 12 girls. What percent of the class are girls?

Fraction -----> Decimal -----> Percent

_______________ , _______________ , _______________

EXAMPLE 2

In the following table, all denominators easily convert to 10, 100, or 1,000. Use the same strategy as in Example 2 to complete the table.

Fraction	Decimal	Percent
$\frac{4}{5}$		
$\frac{28}{50}$		
$\frac{3}{4}$		
$\frac{1}{5}$		
$\frac{9}{10}$		
$\frac{18}{20}$		

EXPLORATION 2: MORE FRACTIONS TO PERCENTS

In the previous exploration, the fractions easily converted to a denominator found in the place value chart, making our job much easier.

Sometimes, however, there is more work to be done. Consider the fraction $\frac{5}{8}$. The denominator, 8, does not have 10 or 100 as a multiple.

Remembering that a fraction is a division problem, we can still make the conversion.

$$\frac{\text{Numerator}}{\text{Denominator}} \quad \frac{5}{8} \quad \frac{\text{Dividend}}{\text{Divisor}}$$

The fraction five-eighths can be written as 5÷8, which we compute below:

$8\overline{)5}$	**Step 1:** Place the dividend inside the division "house", and the divisor outside. This shows we are dividing 5 by 8.
$8\overline{)5.0}$	**Step 2:** Insert a decimal and a zero at the end of the dividend.
0. 8) 5.0	**Step 3:** Begin the process of division. 8 will go into 5 zero times. Write the 0 in the quotient and bring the decimal straight up.
0.6 8) 5.0 4.8 .20	**Step 4:** Continue to divide. 8 goes into 50 six times with a remainder of 2.
0.625 8) 5.0 4.8 0.20 0.16 0.40 0.40 0	**Step 5:** Continue to add zeroes to the dividend to see if the decimal terminates.
$0.625 \cdot 100\% = 62.5\%$	**Step 6:** $\frac{5}{8}$ is equivalent to 0.625 or 62.5%

Now it is your turn! Convert the fraction $\frac{18}{21}$ to a percent.

Hopefully you noticed that the fraction is not in simplest form. Must you simplify the fraction before turning it into a percent? Write your thoughts below:

Remember, a fraction is just a ______________________ ______________________.

Use the space below to convert the fraction to a percent. Write your percent answer on the line below.

____

EXAMPLE 3

As you remember, some decimals do not terminate. For example, $\frac{1}{3}$ is a repeating decimal because:

$1 \div 3 = 0.333\ldots$

To convert a repeating decimal to a percent, we still multiply by 100. Since the decimal never ends, however, we must truncate, or simply cut the decimal off, before multiplying. In the example below, the decimal was only written to the thousandths place.

0.333	3 places
X 100	+ 0 places
33.300	3 places

We see that 0.333 will convert to 33.3%. Knowing that $\frac{1}{3}$ is really 0.333… then, $\frac{1}{3} = 33.\overline{3}\,\%$. Notice the repeating bar is written over the tenths place.

EXAMPLE 4

Complete the table below by converting the fractions with division. Use a repeating bar, if needed.

Fraction	Decimal	Percent
$\frac{5}{6}$		
$\frac{1}{8}$		
$\frac{2}{3}$		
$\frac{1}{9}$		

EXPLORATION 3: PERCENT TO FRACTION

We have converted a lot of fractions to percentages. But, what about changing a percent to a fraction? Recall the meaning of the word "percent".

Percent means
____.

Therefore, 69% means "69 out of 100", or $\frac{69}{100}$.

Likewise, 149% = $\frac{149}{100}$, an improper fraction. We could convert this to a mixed number, ______.

What would 350% look like? _________________________

Now let's try converting a percent to a fraction and a decimal:

79% means 79 ______________________________, or as a fraction____________________.

We have converted the percent to a fraction. Now, use place value to convert to a decimal. Complete the statement below:

79% = _______________ (fraction) = ______________ (decimal)

EXAMPLE 5

In a small bag of 32 pieces of mixed candy, there are 4 pieces of lemon candy.

What percent of the candy is lemon?

What percent of the bag is not lemon?

______% is lemon.

______% is not lemon.

PROBLEMS

1. Complete the following table. Write all fractions in simplest form.

Fraction	Decimal	Percent
$\frac{18}{24}$		
	0.75	
		86%
	1.57	
$\frac{4}{5}$		
	0.9	
		9.8%
	0.005	
$\frac{4}{6}$		
	0.19	
		2.2%

2. There are 28 girls and 20 boys in a math competition.

   What percent of the competition are boys? ____________________

   What percent of the competition are girls? ____________________

3. Brittany is planning a slumber party for her youth group. She took a survey of food choices for dinner. 8 kids chose tacos, 16 chose pizza, 6 chose hamburgers, and 2 could not decide. What percent of the group chose each food:

    ________% chose tacos

    ________% chose pizza

    ________% chose hamburgers

    ________% could not decide

4. Out of 200 parents who voted, only 84 parents voted in favor of school uniforms.

    a. What percent of parents were in favor of school uniforms? ______________

    b. What percent of parents were not in favor of school uniforms? ______________

5. Mr. Garcia has 30 students in his fifth period class. Five-eighths of the students turned in an extra credit assignment.

    What percent of students did not turn in the extra credit assignment? _______________

6. Compare the following numbers using <, >, or =.

a.	$\frac{3}{4}$	☐	0.75	b.	$\frac{29}{200}$	☐	32%
c.	$\frac{7}{8}$	☐	0.9	d.	$\frac{43}{50}$	☐	40%
e.	$\frac{1}{3}$	☐	.33	f.	1.3	☐	100%
g.	$\frac{4}{5}$	☐	85%	h.	0.9	☐	82.5%

7. In Mrs. Cason's Theater Arts Class there are 12 sixth graders, 7 seventh graders, and 6 eighth graders. Of these students, 8 sixth graders, 4 seventh graders, and 5 eighth graders are in the school talent show. Compute the percent of the theater arts students in each grade that will participate in the talent show.

Which grade has the highest percent of participation? ____________________

Which grade has the lowest percent of participation? ____________________

SUMMARY (What I learned in this section)

____

____

____

____

____

____

DECIMAL AND PERCENT REPRESENTATIONS 5

Name:____________________________ Date:________ Period:_______

CHAPTER 5: SPIRAL REVIEW

1. Draw a visual model to show that, in a class of 33 students, $\frac{1}{3}$ of the class is boys and the rest are girls.

2. Write the expanded form for each number. The first one is done for you.
   a. 253. 071 = 200 + 50 + 3 + 0.07 + 0.001
   b. 16.5 = ____
   c. 100.145 = ____
   d. 34.789 = ____
   e. 16.009 = ____

3. Explain how do you convert a decimal to a fraction?

____

____

____

4. Convert each decimal to a fraction or mixed number in simplest form:

Decimal	Fraction or Mixed Number	Decimal	Fraction or Mixed Number	Decimal	Fraction or Mixed Number
0.62		0.409		0.17	
1.75		2.3		4.232	
18.6		67.85		210.18	

5. Evaluate the following expression using the Order of Operations:

$$2^3 \cdot 5 + (34 - 18)$$

____

____

____

6. Add or Subtract the following mixed numbers, and then convert your answer to decimal form:

$7\frac{1}{3} - 4\frac{11}{13} =$	$11\frac{5}{9} - 6\frac{2}{3} =$	$3\frac{1}{2} + 2\frac{8}{13} =$
$12\frac{5}{11} - 1\frac{1}{3} =$	$7\frac{2}{3} - 2\frac{6}{7} =$	$3\frac{5}{16} + 4\frac{1}{3} =$

7. Order the following numbers from least to greatest:

   a. $\frac{3}{5}$, 0.35, 0.099

   ____

   b. 0.601, 0.610, $\frac{5}{8}$

   ____

   c. $\frac{1}{8}$, 1.8, 0.18

   ____

8. Raquel bought 5 notebooks for 99 cents each, 7 binders for $3.99 each, and a pack of pencils for $2.37. Excluding tax, what was the total of her purchases?
   How much change would she get back if she paid with a $50 bill?

   The total cost was ________________, and Raquel received ________________ change back.

9. Show how you can use a factor tree to find the prime factorization of 237:

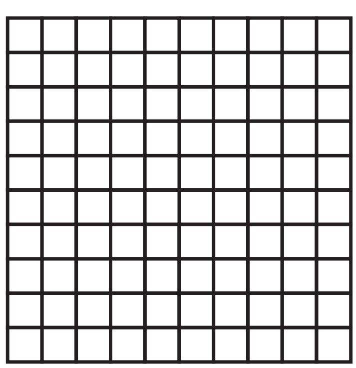
10. Little LiiLii's Lemonade Stand is offering a free cup of lemonade to every 10^{th} customer, a free frozen treat to every 15^{th} customer, and a free cookie to every 6^{th} customer. Identify which customer will be the first to receive all three free items. Explain.

11. Use the following visual models to represent 35%.

Number line

Fraction Strip

10 x 10 grid



EQUATIONS, INEQUALITIES, AND FUNCTIONS 6

Name:___________________________ Date:_______ Period:______

SECTION 6.1 Patterns and Sequences

VOCABULARY

DEFINITION	EXAMPLE
Pattern:	
Term:	
Sequence:	
Constant Difference:	
Arithmetic Sequences:	

Big Idea: How do we find and describe patterns in sequences of numbers?

EXPLORATION 1: PATTERNS AND SEQUENCES

A list of numbers can have interesting characteristics. For example, what pattern do you notice in the following list of numbers?

1, 3, 5, 7, 9, 11, 13, 15,…

Write down a few of your observations about the numbers below:

The number list, or sequence, below has 8 terms listed. What could the next 4 terms be?

1, 3, 5, 7, 9, 11, 13, 15, ______, ______, ______, _______, …

How did you determine the 9th through 12th terms that you wrote above? Explain.

Position	Term
1	3
2	6
3	9
4	12
5	15
6	18

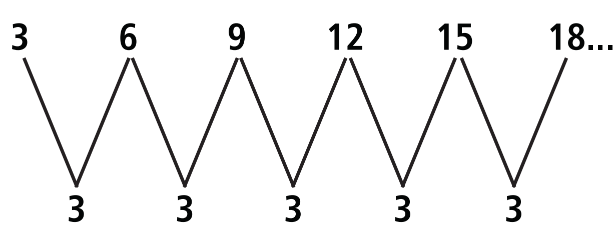
Think about this: If we wanted to know what the twenty-seventh term would be, how could we do that without skip counting each term through the twenty-seventh? Write your ideas below.

Now, let's try using a table to aide us in our discovery. Consider the sequence

3, 6, 9, 12, 15, 18…

We would say the first term is 3, the second term is 6, and so on. Let's make a table with "Position" or "Term number" and "Term" as the heading.

Hint: We can draw in large Vs to show that the term is changing by the same amount with each change of position.



One observation is that the terms are increasing by 3. We say there is a **constant difference**, 3, between the adjacent terms. Lists of numbers with a constant difference are called **arithmetic sequences**. Are any of the other sequences in the exploration above arithmetic sequence? Identify the constant differences.

Another observation that you may have noticed is "horizontally". Is there a pattern that relates the position of the number with the number in the list? The pattern should relate the term number 1 to the term 3, the term number 2 to the term 6, the term number 3 to the term 9, the term number 4 to the term 12 and so on.

We can describe the pattern between the position, n, and the corresponding term as $3n$. Check to see if this expression for the n^{th} term works. The 20th term should be $3 \cdot 20 = 60$.

Write a statement explaining how we could find the n^{th} term.

____

____

Let's try exploring some more sequences using tables.

EXAMPLE 1

Predict the next 3 terms, and then write an expression using the term number to find the n^{th} term.

a. 6, 12, 18, 24,...

Position	Term
1	6
2	12
3	18
4	24
5	
6	
7	

b. 4, 7, 10, 13,…

Position	Term
1	4
2	7
3	10
4	13
5	
6	
7	

c. 2, 5, 8, 11,…

Position	Term
1	2
2	5
3	8
4	11
5	
6	
7	

Expression (a): ________ **Expression (b):** _________ **Expression (c):** _________

d. 2, 6, 10, 14, 18, 22,...

Position	Term
1	2
2	6
3	10
4	14
5	18
6	22
7	
8	
9	

Expression (d): _________

e. 2, 6, 18, 54, 162,…

Position	Term
1	2
2	6
3	18
4	54
5	162
6	
7	
8	

Expression (e): _________

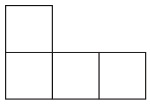
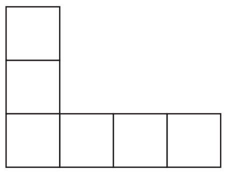
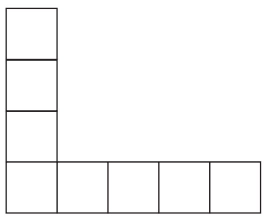
f. 7, 12, 17, 22, 27,…

Position	Term
1	7
2	12
3	17
4	22
5	27
6	
7	
8	

Expression (f): _________

EXPLORATION 2: BLOCK PATTERNS

Some patterns may involve shapes that change in a predictable manner. The example below involves shapes created by blocks. A pattern is observable both as a shape and as a number sequence.

Position	Figure	Term: Number of Blocks
1		4
2		6
3		8
4		
5		
6		

If the pattern continues, determine how many blocks would be in the 4^{th} figure; in the 5th figure; in the 6th figure. What pattern do you observe? Write an expression for the number of blocks in the n^{th} figure.

PROBLEMS

Use tables to describe the number sequences below. Include the next three terms in the sequence. Write an expression for what you think will be the n^{th} term.

1. 10, 100, 1000, 10000, ...
2. 7, 12, 17, 22, 27, 32, ...
3. 3, 7, 11, 15, ...

4. Find the next 3 missing terms, then write the expression for each sequence:

   a. 4, 10, 16, 22, ____ , ____ , ____ , …

   b. 4, 9, 14, 19, ____ , ____ , ____ , …

   Expression: ________________ Expression: ________________

5. Madeline planted a sunflower seed and took measurements each week to chart its growth. The seedling measured 4 inches the first week, 6 inches the second week, and 8 inches the third week. If this pattern continues, how tall will the sunflower be the fourth week? _______________ If this pattern continues, what expression could we use to find the measure of the n^{th} week?________________

6. Julio puts his allowance in the bank each month. He had $75 in January, $150 in February, and $225 in March. If he continues this pattern of saving, how much money will his account hold in June? __________________ in December? _____________

7. A kindergartner is putting the cars of a toy train in a color pattern: red, blue, red, blue… Assuming this pattern continues, what color will the twentieth car be? ____________________

8. Determine if the following sequence is an arithmetic sequence or non- arithmetic sequence. Explain how you know.

   1, 4, 9, 16, 25, …

____

____

9. Create a table to find the next 5 terms in the following sequence. Write the expression you could use to find the n^{th} term:

   12, 22, 32, 42, …

SUMMARY (What I learned in this section)

EQUATIONS, INEQUALITIES, AND FUNCTIONS 6

Name:__________________________ Date:_______ Period:______

SECTION 6.2 EQUATIONS

VOCABULARY

DEFINITION	EXAMPLE
Equation:	
Balanced Equation:	
Solve an Equation:	
Equivalent Equations:	
Subtraction Property of Equality:	

Big Idea: How do you translate a word problem into an equation?

EXPLORATION 1: WRITING EQUATIONS

Let's begin with the sentence "A number is 3 more than 7." You could figure out what this number is with relative ease, but how can you write this mathematically? You may recall that you can use numbers, variables and operations to form expressions. We can now combine these expressions to form a mathematical sentence called an equation. An **equation** is a math sentence with an equality sign, **=**, that relates two expressions.

For example, "three more than fifteen" is an expression. It includes numbers and an operation (addition), but it does not include an equality sign. If we state that "three more than fifteen *is* eighteen" then we have changed our expression to an equation. The word "is" indicates equality. Let's look at an example.

EXAMPLE 1

Translate the sentence "A number is 24 more than 17" into an equation.

SOLUTION

Step 1: We give the unknown number a name, N, and write "N = the number." N is a variable. It represents the number we are trying to find.

Step 2: We translate the sentence into an equation.

A Number	is	24 more than 17
N	=	17 + 24

So the equation form of the sentence is $N = 17 + 24$.

Since $17 + 24 = 41$, we can conclude that $N = 41$. N now represents a known quantity, 41, instead of an unknown quantity. Therefore, we say that we have **solved** the equation for N.

Let's try using the steps above to translate other sentences into equations. Let the variable x represent a number.

a. x less than 22 equals 19 ____________________

b. 4 less than x equals 27 ____________________

c. 6 more than x is 107 ____________________

d. x plus 12 is 30 ____________________

Now, let's look at an example using the number line.

EXAMPLE 2

What number is twice as large as six?

Use the number line to also illustrate the solution.

SOLUTION

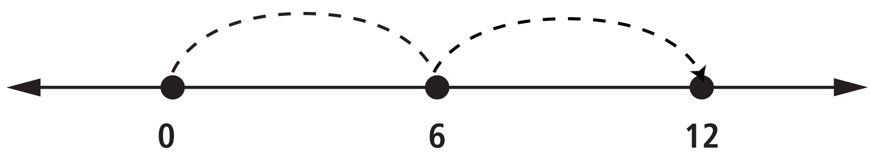
Step 1: We define a variable to represent our unknown number. Let T be a number that is twice as large as six.

Step 2: The statement "A number is twice as large as six" translates as $T = 2 \times 6 = 2 \cdot 6 = (2)(6)$.

When we put the symbols 2 and 6 next to each other with parentheses around each, it is understood that we mean to multiply them. The small dot is also a symbol for multiplication. So $(2)(6) = 2 \cdot 6 = 2 \times 6$. We usually do not use the symbol $\times$, however, since it could be confused with a variable x.

Step 3: We know $2 \cdot 6 = 12$, so $T = 12$.

Step 4: Check. Is 12 a number that is twice as large as 6? Yes.



Now it's your turn. Use the line below to create a number line model the for the next word problem. Write an equation to find the solution.

Pat purchased a dozen doughnuts. The number of milk cartons Terry purchased is twice the number of doughnuts Pat purchased. How many milk cartons did Terry purchase?

What equation did you write to solve the problem? ____________________________

How many milk cartons did Terry purchase? _______

EXAMPLE 3

Translate the sentence "A number is 2 less than four times 10" into an equation and solve for the unknown variable. Does your answer make sense?

Step 1: Select a variable to represent your unknown value. Write it: _____________

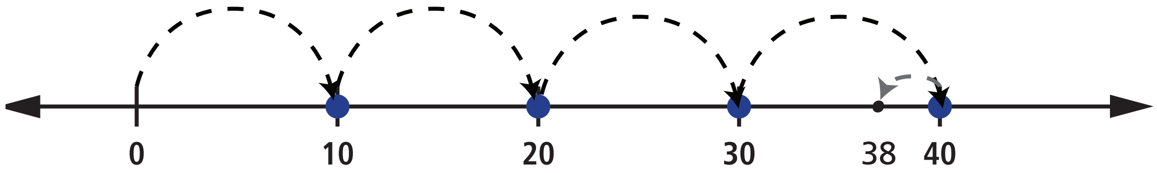
Step 2: Now write an equation for the statement in EXAMPLE 3: ___________________

Step 3: The left side of your equation equals ________ and the right side equals _______. Is your equation balanced?

Step 4: Check your equation by substituting in the value that you found: ______________________ Does your answer make sense? ______________

Check your work on the number line.

Notice on the number line below that 4 times 10 is 40 and 2 less than 40 is 38.



EXPLORATION 2: CHARTING THE PROCESS

We have seen how we can use numbers and variables to translate problems into equations. Consider the problem, "Jeremy is 9 years old. In how many years will Jeremy be 15 years old?"

How might you begin this problem? Did you define a variable? If so, how did you use this variable?

Here is a step-by-step approach. Do your steps resemble the following?

Step 1: Define your variable

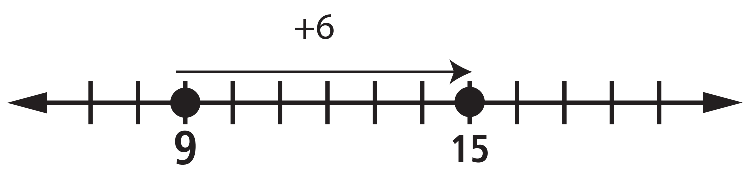
Y = the number of years it takes for Jeremy to reach 15.

Step 2: Translate the problem into an equation

We know that 15 is Y more than 9, so we write $15 = 9 + Y$, an equation with one variable, Y.

Step 3: Solve for the unknown variable

If you look on the number line, you'll notice you have to move right 6 units to go from 9 to 15. So $Y = 6$.



Step 4: Check your answer

Substitute $Y = 6$ into the original equation to see that $15 = 9 + 6$.

Using the 4 step process discussed above, create a poster showing how the steps should be used to solve a problem. Begin by writing the problem and continue by showing how each step helps you solve the problem.

Another way to visualize an equation is with a balance scale.

This is a balance scale:

When we put a weight on one side of the scale, we must place the same weight on the other side in order for the scale to be balanced. If a scale is balanced and equal weights are added or subtracted from both sides of the scale, the scale will remain balanced.

In much the same way, an equation is a statement that two expressions are equal. We can think of the expressions on each side of the equality sign as representing the weight placed on each side of a balanced scale. When we add or subtract the same amount from each side of the equation, the equation will remain **balanced**.

EXAMPLE 4

If Jeremy were three years older, he would be the same age as his twelve-year-old sister. What is Jeremy's age? We let J be Jeremy's age, and translate the sentence into an equation as follows:

Jeremy's age	**three years older**	**same age as**	**twelve-year-old sister**
J	$+ 3$	$=$	12

Now we have the equation $J + 3 = 12$. The unknown is J, Jeremy's age. Pictorially, this sentence says that $J + 3$ is the same as 12, which we can show on a balance scale:

In order to solve this equation for J, we must find what balances J. To do this, we remove three "blocks" from each side of the balance scale:

This is what we have left:

We can express this algebraically as follows:

Because we have now solved for *J*, we can go back and check the solution. Substituting J = 9 into the original equation J + 3 = 12 gives us 9 + 3 = 12, which is correct. Jeremy's age is 9.

Write an equation to solve the next problem and model it on a balance scale.

EXAMPLE 5

If Wesley finds 5 more marbles, he will have the same number of marbles as John. John has 11 marbles. How many marbles does Wesley have?

EXPLORATION 3: SUBTRACTION PROPERTY OF EQUALITY

The rule we are using to solve these problems is called the **subtraction property of equality** because in each, we are subtracting the same number (removing the same number of blocks) from both sides of an equation. The new equation we obtain is said to be **equivalent** to the original equation because the two equations have the same solution: any value for a variable that makes one of the equations balance will make the other balance as well.

Definition 6.2: Subtraction property of Equality
If $A = B$ then $A - C = B - C$.

Consider the sequence 3, 4, 5, 6, ... We see that the term in a particular position, n, is $n + 2$. We can write this as the term = position + 2 or $t = n + 2$, where we let the variables t = term and n = position. According to this pattern or "rule", the 20th term would be 22 and the 200th term would be 202. Suppose you were given the term 187. Can you determine which position this number would be in the list above?

Remember that t = ___________, and 187 is the term. Symbolically, we could write t = ____________.
If we rewrite the equation substituting 187 for t, how can we determine n?
____ How could we use the subtraction property to solve?
____

Finding the value that makes an equation true is referred to as **solving an equation**. Solving equations is a very important part of doing algebra and subtraction property is an important tool for solving equations.

EXPLORATION 4: EQUATIONS AND SEQUENCES

The total cost of a meal for three people is \$51. If the three people agree to split the cost equally, what would each person's cost be? Write two equations that could be used to model the problem. You do not have to solve the problem. Use x to represent the cost each person pays. ____________________________

You may have found that one equation is $3 \cdot x = 51$. Another equation could be written as $x = \frac{51}{3}$. You may recall this important connection between multiplication and division. We will talk more about this when we multiply by fractions in the next chapter.

We can relate the idea above with the sequence 3, 6, 9, 12, 15... Suppose we ask the question, "What position is 51 in this list?" We write this question mathematically as $3 \cdot x = 51$, where we let x represent the position the term 51 occupies in the list. From above, $3 \cdot x = 51$ is equivalent to $x = \frac{51}{3} = 17$. We conclude that 51 occupies the 17th position in the list above.

Try using the same strategy to solve the word problem below.

You and four friends have dinner at a nice restaurant. The bill comes to \$135, which you agreed to split evenly.

Write two equations that could be used to model the problem:

____ or ____

If we think of the sequence 5, 10, 15, 20, … , we want to know what position 135 is in the list. Use an equation from above to solve. 135 occupies the ________ position in the sequence.

PROBLEMS

1. Translate the sentences below where *x* is a number.

   x less than 10 equals 8.

   a. ______________________

   10 less than *x* equals 8.

   b. ______________________

2. Fill in the blanks to complete the steps to solve an equation.

   a. Step 1: Define your _____________

   b. Step 2: Translate the problem into an _______________

   c. Step 3: Solve for the ______________ _______________

   d. Step 4: _______________ your answer

3. Colton purchased half of a dozen candy bars from the school fundraiser. Robin purchased 5 times that amount. Write an equation to find how many candy bars Robin purchased. Remember to follow the steps listed above.

4. Write an equation for each scenario below and solve to find the unknown value.

   a. Martin has $30. How much more does he need if he wants a total of $113?

   b. If BJ will be 19 in 7 years, how old is BJ now?

   c. Ben is 16 years old. In how many years will he be 34?

   d. Brooke is 59 inches tall. How tall will she be after a 4-inch growth spurt?

5. Alison has a certain number of gel pens, let g represent the number of pens that Alison has. Pablo has 7 more pens than Alison, and Raquel has three times as many pens as Pablo. How many pens does Raquel have if Alison has:

   a. 5 pens? ___________ b. 10 pens?___________ c. g pens? ___________

6. Write a story problem for this equation: $x - 32 = 75$

   ____

   ____

7. Solve the following equations using the 4-step process:

   a. $m - 16 = 20$

   c. $f - 3.01 = 12.4$

   b. $16.2 + s = 45.095$

   d. $14.7 + t = 14.9$

SUMMARY (What I learned in this section)

EQUATIONS, INEQUALITIES, AND FUNCTIONS 6

Name:___________________________ Date:_______ Period:______

SECTION 6.3 Equations and Inequalities on Number Lines

VOCABULARY

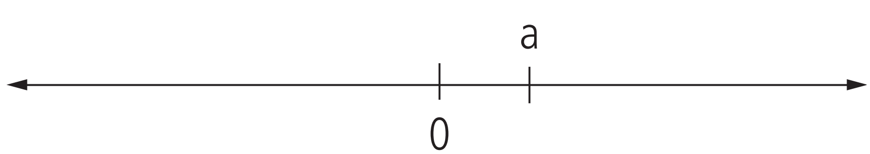
DEFINITION	EXAMPLE
Inequality:	

Big Idea: How do we solve an equation on the number line?

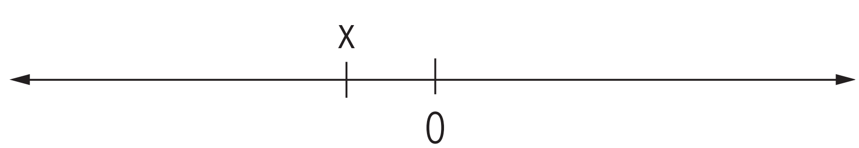
EXPLORATION 1:

Suppose a and x are numbers located on the number line as seen below. Locate and label the points that represent the indicated numbers. Use string to act out how you determine your answer.

1. Plot points that represents each of the following: 2a, 3a, -a, -2a, -3a



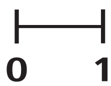
2. Plot points that represents each of the following: 2x, 3x, -x, -2x, -3x



3. Compare the results from parts 1 and 2. What do you notice?

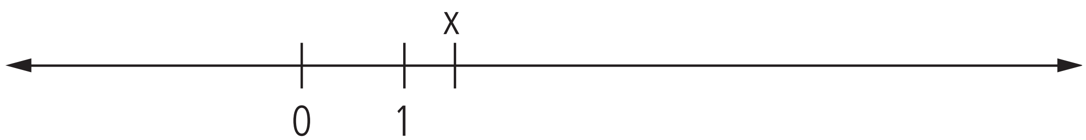
EXPLORATION 2:

Part A: Suppose x is a number that is located on the number line as seen below. Locate and label the points that represent the indicated expressions. The numbers 0 and 1 are also labeled. The length of the line segment below is 1:

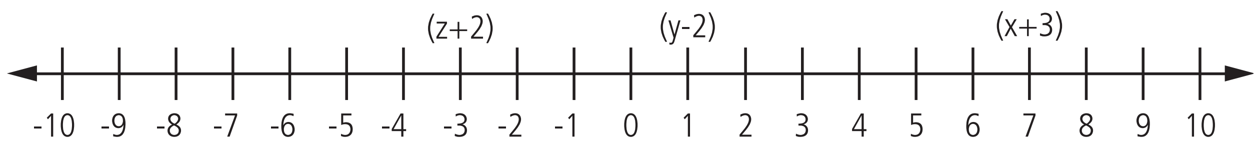


Plot a point that represents each of the following expressions:

x + 1, x + 2, x + 3, x – 1, x – 2



Part B: Suppose we know the location of each of the expressions as indicated on the number line below. Find the locations for x, y, and z. Explain how you locate each of these points on the number line.



PROBLEMS

Use the number line to solve each of the following equations:

a. $x + 3 = 5$

b. $y + 5 = 2$

c. $z - 4 = 2$

d. Discuss how solving these equations on the number line compares with the balance scale method.

Recall that an equation is a statement that two expressions are equivalent. A statement that one expression is always less than (or greater than) another is called an **inequality**.

EXAMPLE 1: Translate the following into mathematical expressions

1. The number of apples, A, consumed is more than twice the number of bananas, B.

2. Jack's age, J, is less than 40 years.

EXAMPLE 2:

Draw a number line and represent the set S of all numbers x such that $x < 3$.

Draw a number line and represent the set T of all numbers x such that $-2 \leq x$.

If we start with an inequality, such as $x + 3 < 5$, determine what numbers x satisfy this inequality?

Represent the inequality on the number line.

SUMMARY (What I learned in this section)

EQUATIONS, INEQUALITIES, AND FUNCTIONS 6

Name:___________________________ **Date:**_______ **Period:**______

SECTION 6.4 Functions

VOCABULARY

DEFINITION	EXAMPLE
Function:	
Output:	
Input:	
Domain:	
Range:	
Graph (of a function):	
Function Notation:	

Big Idea: What is a function? How do we find the rule of a function and use it to create a table and a graph?

EXPLORATION 1: USING TABLES TO REVEAL A PATTERN

Sarah builds model airplanes. She makes two airplanes each day. How many airplanes will she make in 4 days? ________________10 days? ____________________. Organize the information to reveal a pattern in the number of airplanes she makes in a given number of days.

How did you organize the information in the exploration above?

Do you see a pattern in the number of airplanes she can make in a given number of days?

One way to organize such information is to build a table such as the one to the right. Notice that the first column is the number of days, and the second column is the total number of airplanes that Sarah can make in the corresponding number of days

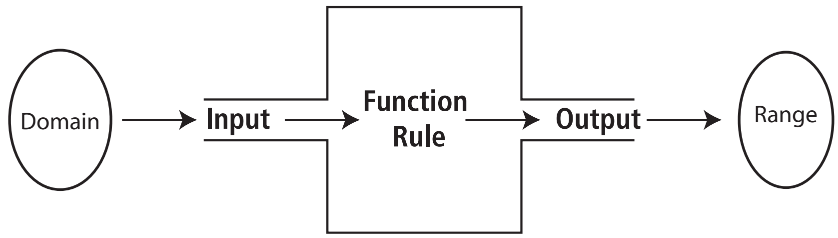
What do you notice about this table?

Why is this a good way to organize the information?

Days	Total Number of Planes	Ordered Pairs
0		
1		
2		
3		
5		
10		
x		

This is an example of a function. There is a rule, or function, to determine how many planes Sarah has produced based on the number of days she has worked. You can think of a function as a machine with inputs and outputs. The input is the number of days Sarah has worked. The output is the number of planes produced.

Let's think a little bit more about the function machine just mentioned by first viewing an example of what it might look like.



Let's say the Function Rule is 5*x*-1. Think of a number, or value of *x*, to input into the machine. We'll again organize our information on a table.

Input (Domain)	Function Rule $5x-1$	Output (Range)	Ordered Pair
1	5 (1) – 1	4	(1,4)
2	5 (2) – 1		
3			
4			

To continue, substitute the value of 1 in for *x*, and find what is 5 (1) –1. Write your answer in the output column. We could also write this as an ordered pair, (1, 4), meaning that when the *x* value is 1, the *y* value is 4. If the input is x = 1, could the *y* value be any other number? ______________

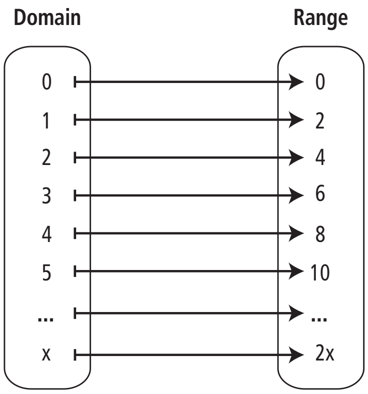
Explain your answer. ____

Continue inputting the values of *x* to find the corresponding output and record it on the table.

Now that we have an idea of what a function does, let's go ahead and make a more formal definition.

Definition 7.1: Function
A **function** is a rule that assigns to each member of a set of inputs, called the **domain**, a member of a set of outputs, called the **range**.

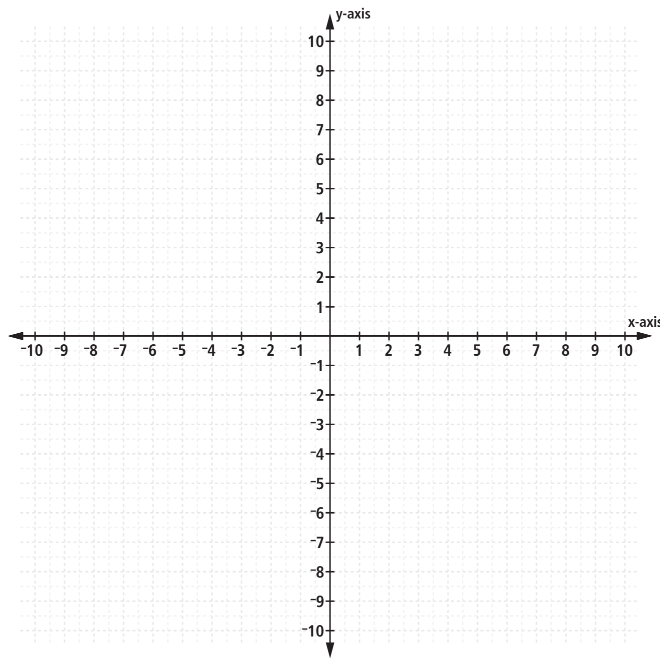
Let's look again at the first example involving Sarah. The domain is the set of nonnegative integers 0, 1, 2, 3,... and the range is the set of even nonnegative integers 0, 2, 4, 6, … . In general, the domain = the set of inputs and the range = the set of outputs, as shown in the illustration below.



Notice that a function produces input and output pairs of numbers.

Input	Function Rule	Output
x	Multiply input by 2 to get the output $y = 2(x) = 2x$	
0	$y = 2(0)$	
1	$y = 2(1)$	
2	$y = 2(2)$	
3	$y = 2(3)$	
x	$y = 2(x)$	

Let's call Sarah's function *F*. From the table and the picture above, we can see that (0, 0), (1, 2), (2, 4), (3, 6), and (5, 10) are some of the pairs that belong to the function *F*, where the *x* or first-coordinate is the input and the *y* or second-coordinate is the output. In other words, the ordered pairs are of the form (input, output) or (x, y). Each pair of numbers can be thought of as a point on the coordinate system, so we can also talk about the **graph** of a function. The graph of the function is the pictorial representation of the function.



In mathematics, a **notation** is a technical system of symbols used to represent unique objects. We can write "The function *F* pairs the number 1 with 2" symbolically as "$F(1) = 2$." We read this as "*F* of 1 equals 2." This means *F* sends the input 1 to the output 2. Similarly, because the function *F* pairs the number 2 in our domain with the number 4 in the range to give us the pair (2, 4), we write "$F(2) = 4$."

So $F(x)$ = the number of planes that can be produced in *x* days. We can express this rule in general as $F(x) = 2x$.

EXPLORATION 2: MORE FUNCTION PRACTICE

Consider the following tables of numbers that describe a function.

Function F

Input	Function Rule	Output
x		*y*
1		13
2		14
5		17
7		19
8		20

1. Describe in words all the patterns that you observe between the input and the output of Functions F.
2. Use your pattern rule to determine the output if the input is 10.
3. Write an expression for your output *y* if the input is *x*.
4. Find the output for the function F if the input is 30.
5. Graph the points from the table.

Function G

Input	Function Rule	Output
x		*y*
0		0
1		3
2		6
3		9
4		12

1. Describe in words all the patterns that you observe between the input and the output of Functions G.
2. Use your pattern rule to determine the output if the input is 10.
3. Write an expression for your output *y* if the input is *x*.
4. Find the output for the function *G* if the input is 30.
5. Graph the function and connect the sample points from your table.

EXPLORATION 3: FUNCTIONS ON YOUR OWN

The functions have rules for the input, x, and the output, y. Make a table for each function below to indicate the outputs for the inputs, 0, 1, 2, 3, 4, 10, 15. Make sure your table includes a column that shows how you got the output.

1. The function rule for R is given by the equation $y = 4x + 7$.
2. The function rule for S is given by the equation $y = 4$.
3. Use the R function to determine the input if the output is 55.

PROBLEMS

1. Consider the function J given by the rule $y = 3 + x$. Use the table below to determine the outputs that correspond to the inputs given below.

Input (Domain)	Function Rule $y = 3 + x$	Output (Range)
0		
1		
2		
3		
4		

2. Fill in the table using the Function Rule stated.

Input (Domain)	**Function Rule $y = 2x - 3$**	**Output (Range)**
5		
6		
8		
9		
11		

3. Find what rule would make the table below true. Write the rule in the Output column beside the input, *x*.

Input, x (Domain)	**Output, y (Range)**
0	**100**
1	**99**
3	**97**
4	**96**
6	**94**
x	

4. Colton incorrectly filled out the table below for the rule $y = 5+x$. Check his work to find the error.

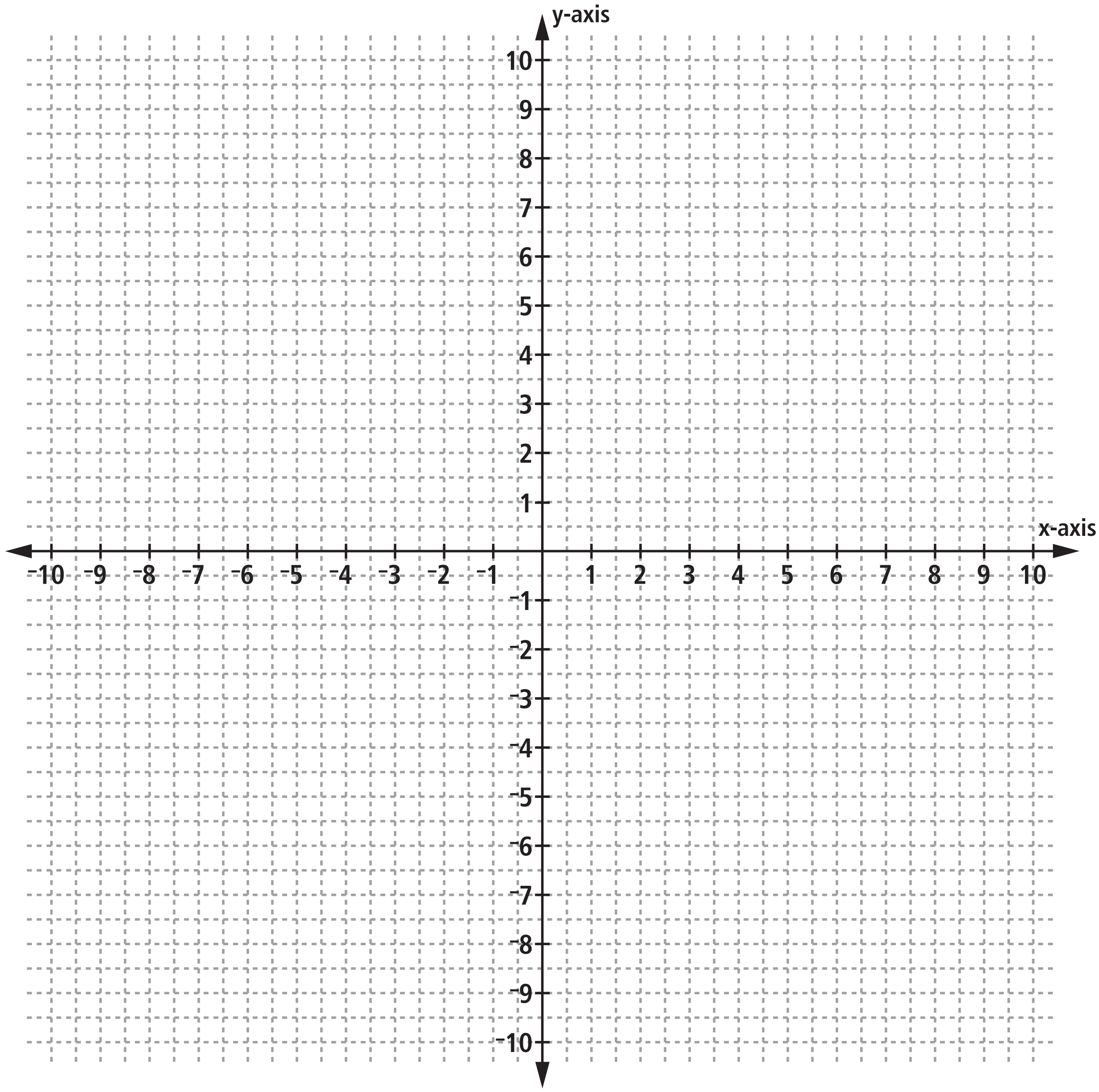
Input (Domain)	**Output (Range)**
0	**0**
1	**5**
2	**10**
3	**15**
4	**20**

What mistake did Colton make? ____

Draw a new table beside the table above and fill it in with the correct values.

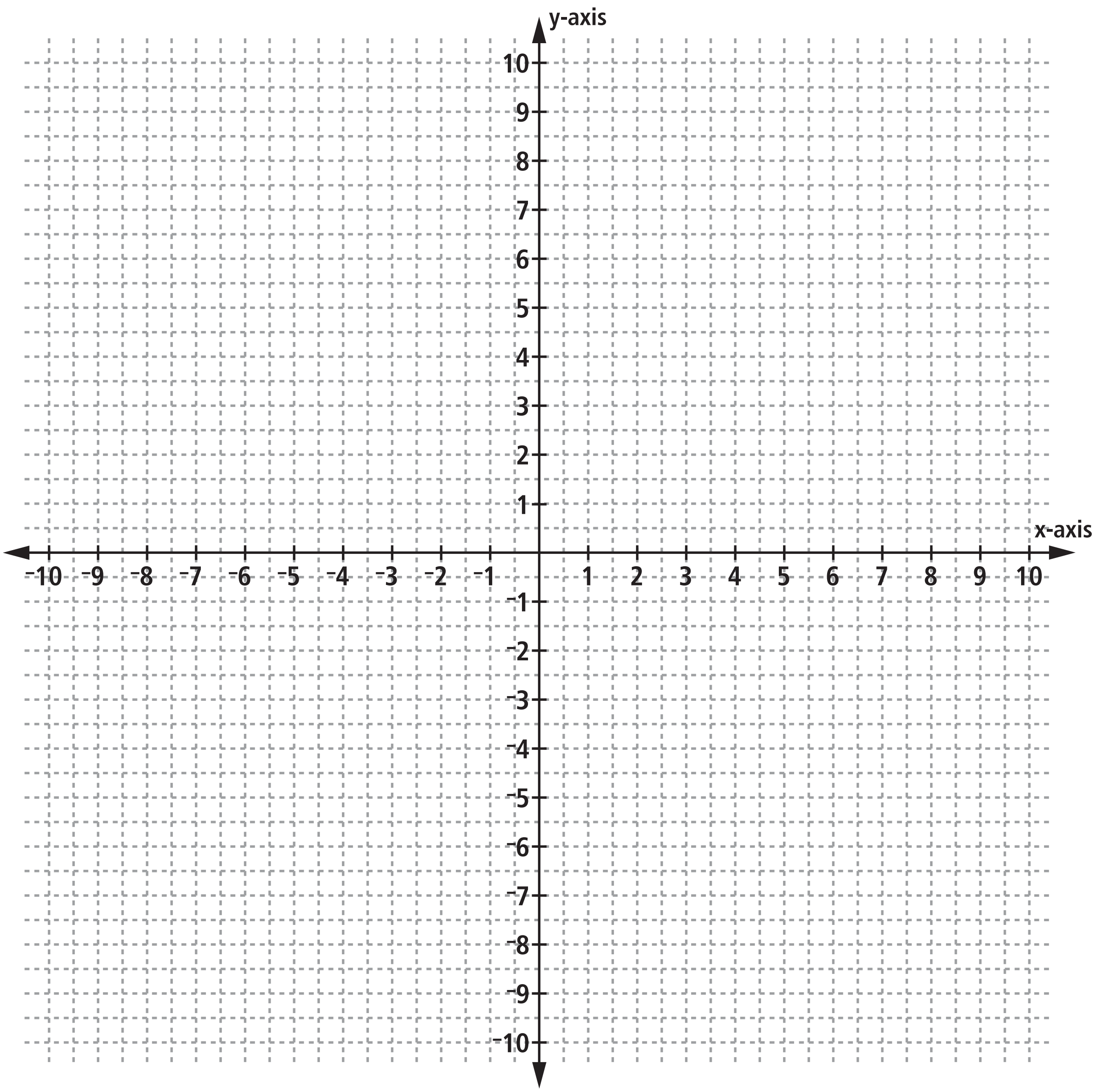
5. Complete the table below according to the rule given. Then plot the values in the coordinate plane below.

Input (Domain)	Function Rule $y = x + 1$	Output (Range)	Ordered Pair (x, y)
0			
1			
2			
3			
4			



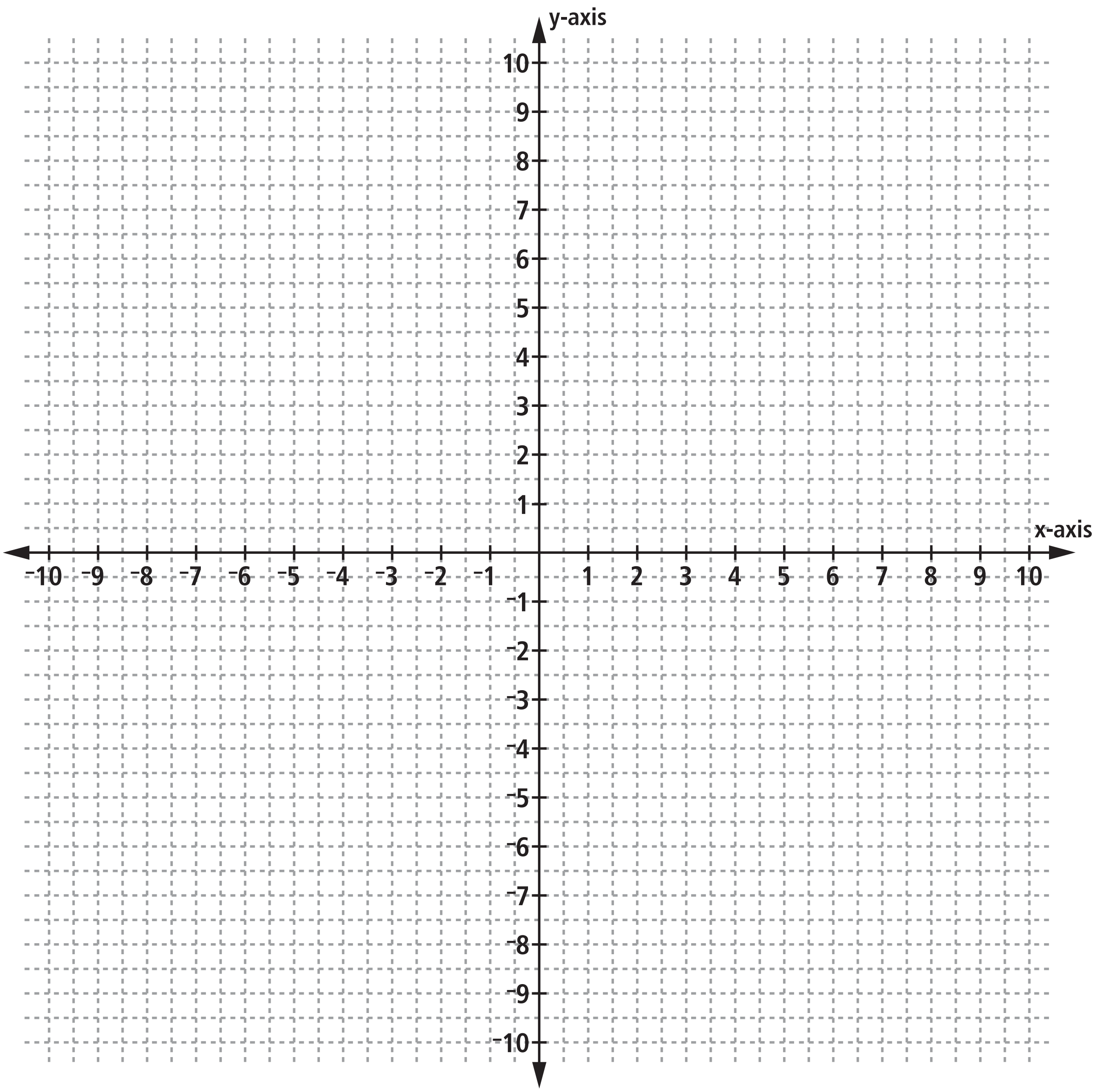
6. Complete the table below according to the rule given. Then plot the values in the coordinate plane below. Remember when you write a number beside a variable, such as $0.5x$, it means to multiply the number times the variable.

Input (Domain)	Function Rule $y = 0.5x$	Output (Range)	Ordered Pair (x, y)
0			
1			
2			
3			
4			



7. Complete the table below according to the rule given. Then plot the values in the coordinate plane below.

Input (Domain)	Function Rule $y = 8 - 2x$	Output (Range)	Ordered Pair (x, y)
0			
1			
2			
3			
4			



SUMMARY (What I learned in this section)

EQUATIONS, INEQUALITIES, AND FUNCTIONS 6

Name:____________________________ **Date:**________ **Period:**_______

CHAPTER 6: SPIRAL REVIEW

1. Write an equation for each statement below:
   a. 3 more than a number times 5 is 48 ___________________________________
   b. A number divided by 2 is 16 ____
   c. The quotient of 8 and a number is 4 __________________________________
   d. 14 less than a number is 20 ____

2. Katy is making bags of party favors for her birthday party. She has 27 bottles of fingernail polish, 15 files, and 18 bottles of lotion. What is the greatest number of identical party favor bags Katy can make?

   Katy can make __________ bags of party favors. Each bag will have _____ bottles of fingernail polish, ______ files, and _____ bottles of lotion.

3. Convert each of the improper fractions to a mixed number. Make sure all fractions are in simplest form.

Improper Fraction	Mixed Number	Improper Fraction	Mixed Number	Improper Fraction	Mixed Number
$\frac{38}{4}$		$\frac{56}{13}$		$\frac{118}{9}$	
$\frac{29}{3}$		$\frac{82}{11}$		$\frac{39}{4}$	
$\frac{18}{2}$		$\frac{45}{23}$		$\frac{46}{8}$	

4. Evaluate the following expressions:

   a. $16 \cdot (3^3 - 7) \div 2$ ____________________ ____________________ ____________________ ____________________

   b. $7^2 - 6 \cdot 2 + 21$ ____________________ ____________________ ____________________ ____________________

5. Add or Subtract the following whole numbers, fractions, or mixed numbers:

$\frac{3}{8} + \frac{5}{9} =$	$1\frac{5}{9} - \frac{1}{2} =$	$4 - 2\frac{5}{16} =$
$\frac{8}{13} + \frac{1}{3} =$	$\frac{11}{13} - \frac{1}{2} =$	$6\frac{5}{9} + 4\frac{5}{9} =$

6. Consider the sequence below:

   5, 9, 13, 17, 21, …

   The next two terms in the sequence would be: __________, __________

   Write the rule to find the nth term: ______________________________

7. Reese makes glitzy cell phone covers for her friends. Each cover requires 63 crystals for decoration. Complete the table below to show the relationship between the number of phone covers made and the number of crystals used. Then write an equation that could be used to find *c*, the number of crystals needed in terms of *p*, the number of cell phone covers made.

Number of phone covers, *(p)*	**Number of Crystals, *(c)***

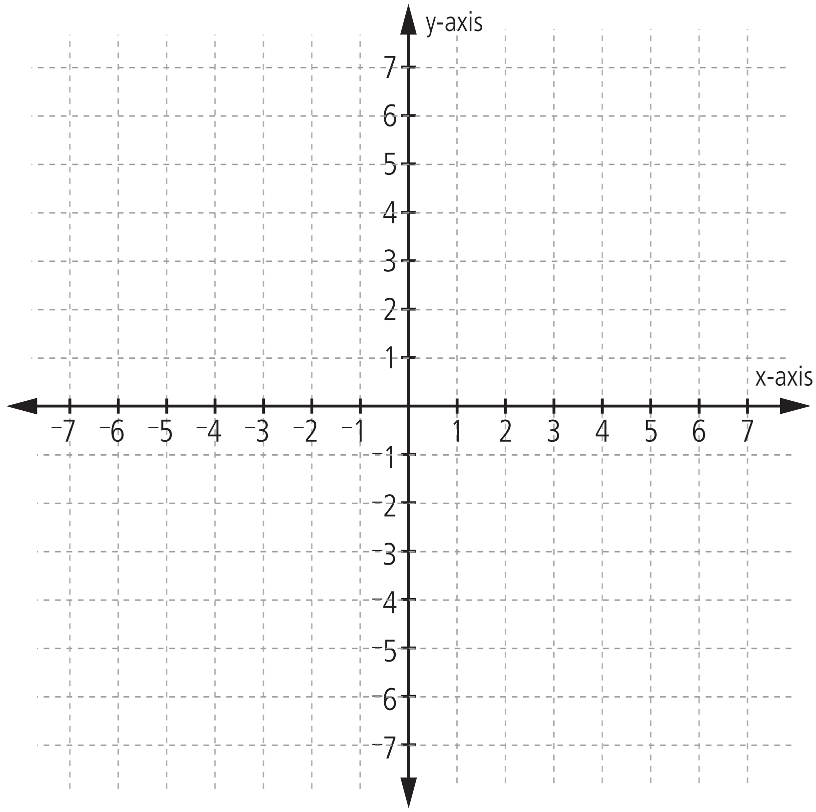
Equation: ____

8. Solve the following equations:

$7 + p = 211$	$65 - x = 14$	$3t = 63$
$46 + k = 82$	$v - 36 = 40$	$15 \div f = 3$

9. Follow the function rule to complete the table and graph the function:

Input, *x*	**Function Rule $y = 3x + 2$**	**Output, *y***	**Ordered Pairs *(x, y)***
0			
1			
2			
5			
7			
x			



10. Consider the following pattern of inputs and outputs. Write a rule that gives the output, *y*, in terms of the input, *x*.

Input, *x*	**Output, *y***
3	**12**
4	**17**
5	**22**
6	**27**
7	**32**

Rule: ______________________________

RATES, RATIOS, AND PROPORTIONS 7

Name:________________________________ **Date:**________ **Period:**______

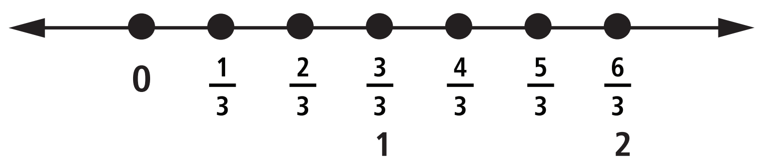
SECTION 7.1 Multiplying Fractions

Big Idea: How do we multiply fractions?

Just as we used the linear model to understand multiplication of integers, we will begin our discussion of multiplying fractions by exploring this process on a number line.

EXPLORATION 1: MULTIPLYING FRACTIONS ON THE NUMBER LINE

What would $\frac{1}{3}$ of 6 be? In other words, what is the product $\frac{1}{3} \cdot 6$? Remember that we multiply by using the first factor as the length of the jump and the second factor as the number of jumps. Illustrate the process on the number line below and represent the product in words.



With each jump, the frog will advance ______ on the number line. The frog will make six jumps. Therefore, the product of $\frac{1}{3} \cdot 6$ is ___________________________________.

EXAMPLE 1

Jane has $\frac{1}{2}$ a yard of ribbon and needs to cut $\frac{1}{3}$ of its length. To do this, she needs to know what $\frac{1}{3}$ of $\frac{1}{2}$ is. Write the appropriate multiplication problem in the space below:

____

Use the number line below, to calculate $\frac{1}{3} \cdot \frac{1}{2}$. Scale carefully.

If each jump is $\frac{1}{3}$, and the frog makes $\frac{1}{2}$ of a jump, it travels _____ of a yard.

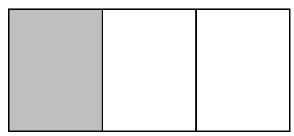
PROBLEM 1

Emma runs only $\frac{1}{2}$ of the $\frac{3}{4}$ mile track, what fraction of a mile does Emma run. Use the number line to calculate.

EXPLORATION 2: MULTIPLYING FRACTIONS WITH THE AREA MODEL

With the linear model it is important to be very exact when drawing the picture. To see the advantage of using the area model, let's look again at the earlier problem: $\frac{1}{3} \cdot \frac{1}{2}$.

Begin the process by using the rectangle below, which models $\frac{1}{3}$ as a shaded part of the whole rectangle with an area of 1:



Now we can represent $\frac{1}{2}$ of the shaded area by cutting the rectangle in half horizontally. To model this, we will shade the upper half of the rectangle in another color.

You can see that one of the pieces from the second cut is doubly shaded. This portion of the rectangle represents $\frac{1}{2}$ of the original $\frac{1}{3}$ rectangle. What part of the whole rectangle is the double-shaded area?

____

EXAMPLE 2

Translate $\frac{1}{2}$ of $\frac{1}{5}$ into a multiplication problem. __________________________________

Now, use the rectangle below to draw the corresponding area model to find the product.

What part of the whole rectangle is the double-shaded area? ________________________

We see that the product of $\frac{1}{2} \cdot \frac{1}{5}$ is ______________.

Make a conjecture about a rule for multiplying unit fractions: $\frac{1}{m} \cdot \frac{1}{n} =$ ________

EXAMPLE 3

What is $\frac{2}{5}$ of $\frac{2}{3}$? First, let's translate the statement into a multiplication problem.

____

Now, use rectangle below to draw the area model and find the answer.

What part of the whole rectangle is the double-shaded area? ________________________

We see that the product of $\frac{2}{5} \cdot \frac{2}{3}$ is ______________.

Make a conjecture about a rule for multiplying fractions: $\frac{a}{b} \cdot \frac{c}{d} =$ ________

EXPLORATION 3: APPLYING THE RULE FOR MULTIPLYING FRACTIONS

The linear and area models are useful in helping us understand the process for multiplying fractions. However, as you begin to study ratios and rates in the next section, you will find that being able to apply the rule for multiplying fractions is the most efficient to solve these problems.

Let's try applying our rule, to solving a problem.

Consider $\frac{3}{5} \cdot \frac{4}{7}$. Since we know that $\frac{a}{b} \cdot \frac{c}{d} = a \cdot \frac{c}{b} \cdot d$, let' try rewriting the problem showing our work along the way.

$$\frac{3}{5} \cdot \frac{4}{7} = 3 \cdot \frac{4}{5} \cdot 7 = \_\_\_\_\_\_\_\_$$

Now try solving a few problems on your own, following our rule (without referring to a number line or an area model). Remember to show your work AND write your final answer in simplest form.

a. $\frac{1}{3} \cdot \frac{1}{6} =$	b. $\frac{5}{8} \cdot \frac{1}{2} =$	c. $\frac{1}{4} \cdot 100 =$
d. $\frac{3}{9} \cdot \frac{1}{3} =$	e. $26 \cdot \frac{1}{2} =$	f. $\frac{6}{16} \cdot \frac{2}{3} =$

PROBLEMS

1. Create an area model to illustrate and solve the following products.

   a. $\frac{2}{3} \cdot \frac{1}{3} =$ ____________

   b. $\frac{1}{4} \cdot \frac{1}{3} =$ ____________

   c. $\frac{1}{3} \cdot \frac{1}{6} =$ ____________

   d. $\frac{2}{7} \cdot \frac{1}{3} =$ ____________

2. Compute the following products. Show your work and simplify if needed:

   a. $\frac{7}{9} \cdot \frac{1}{6} =$

   b. $\frac{7}{10} \cdot \frac{1}{7} =$

   c. $16 \cdot \frac{1}{4} =$

   d. $\frac{4}{8} \cdot \frac{1}{3} =$

   e. $\frac{2}{3} \cdot \frac{9}{12} =$

   f. $\frac{1}{6} \cdot 120 =$

3. A pancake recipe calls for $\frac{1}{8}$ cup sugar. Lauren is making $\frac{1}{2}$ of the recipe. How much sugar will she need? Write an equation and show your work to solve the problem.

   Lauren will need _________ cup sugar.

4. Gavin spent $\frac{3}{4}$ hour practicing piano. He spent $\frac{1}{3}$ of that time practicing his scales. What portion of an hour did Gavin spend on his scales? Write an equation and show your work to solve the problem.

   Gavin spent ________ of an hour practicing his scales.

5. Tyrone takes $\frac{2}{5}$ of a pizza to school for lunch. He shares $\frac{1}{3}$ of the pizza with his classmate. How much of the pizza is left for Tyrone? Write an equation and show your work to solve the problem.

   Tyrone had ________ of a pizza remaining.

6. When Mr. Soto asked for volunteers to participate in the Math Fair, $\frac{2}{3}$ of his students signed up. If he has 24 students in his class, how many students signed up for Math Fair? Write an equation and show your work to solve the problem.

   _____ students signed up to participate in Math Fair.

7. Mr. Reyes is planning a garden. He will use $\frac{2}{3}$ of his garden to plant vegetables and will use the remaining space to grow berries. His garden is 300 square feet.

   a. If $\frac{1}{8}$ of the vegetable garden is used to grow beans, what portion of the total garden is planted with beans? Write your answer in simplest form.

   b. If Mr. Reyes' berry patch is $\frac{1}{4}$ blackberries and $\frac{3}{4}$ strawberries, what portion of the total garden is planted with strawberries? Write your answer in simplest form.

   c. What is the area of the strawberry patch?

SUMMARY (What I learned in this section)

Name:_________________________ **Date:**_______ **Period:**______

SECTION 7.2 DIVISION OF FRACTIONS

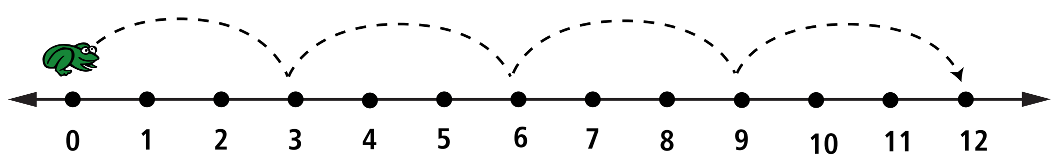
VOCABULARY

DEFINITION	EXAMPLE
Reciprocal:	
Multiplicative Inverse:	

Big Idea: How do we divide fractions?

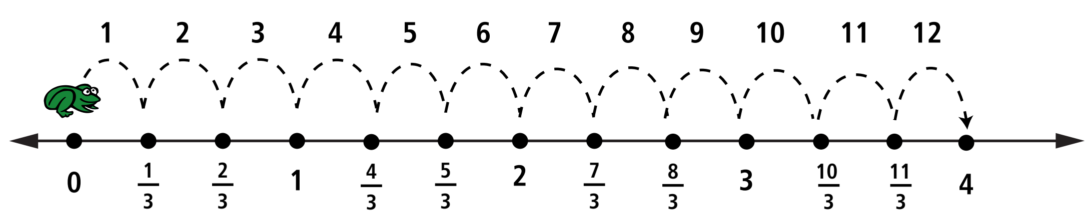
EXPLORATION 1
Melinda wants to cut a 4-yard fabric into $\frac{1}{3}$ - yard strips. How many strips will she have? Explain how you reached your conclusion.

Just as with a whole number division problem such as cutting a 12-yard fabric into 3 yard strips, you can use a linear model to show that there are 4 strips possible.



That is $12 \div 3 = 4$.

You can also use a linear model to represent the problem in exploration 1.



We see that $4 \div \frac{1}{3} = 12$.

PROBLEM 1

Liz has 4 pounds of jellybeans. She plans to make little party bags containing a $\frac{1}{2}$ pound of jellybeans. How many party bags can she make? Explain using a linear model.

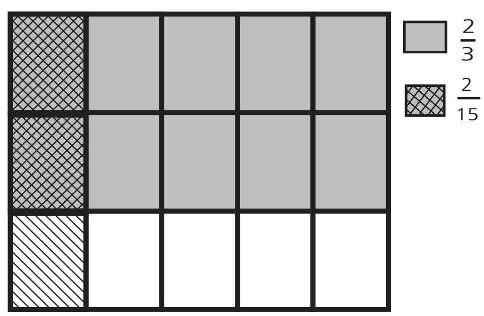
EXPLORATION 2

Chuck has two-thirds of a pan of brownies and shares it evenly among 5 friends. What fraction of the pan of brownies does each friend receive?

Explain how you reached your conclusion.

One way to think about the problem is as a division of $\frac{2}{3} \div 5$. Another approach is to think of each of Chuck's 5 friends receiving $\frac{1}{5}$ of the brownies or as a multiplication problem, $\frac{2}{3} \cdot \frac{1}{5}$.

An area model of this problem can be represented as we have below:



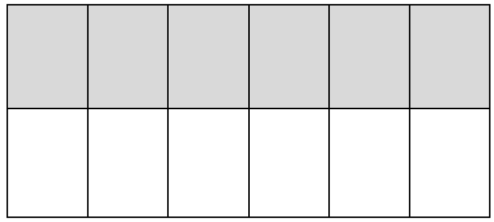
$\frac{2}{3} \div 5 = \frac{2}{3} \cdot \frac{1}{5} = \frac{2}{15}$ or the darker cross-hatched shaded region.

PROBLEM 2

Barbara has $\frac{1}{2}$ of a pan of brownies and shares it evenly among 6 friends. What part of the pan of brownies does each friend receive?

You can use the area model to show and explain how you reached your conclusion.

$\frac{1}{2}$

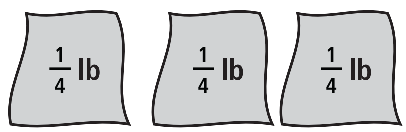


Notice each friend received $\frac{1}{12}$ of the original pan of brownies.

Look at a similar problem but with fractional quantities:

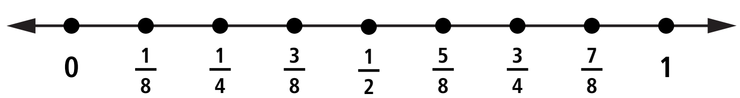
How many $\frac{1}{4}$-pound bags does it take to pack $\frac{3}{4}$ pounds of sand? In other words, what is $\frac{3}{4} \div \frac{1}{4}$?

Using a repeated subtraction model, make 3 equal parts. With the first $\frac{1}{4}$-pound bag, $\frac{3}{4} - \frac{1}{4} = \frac{1}{2}$ pounds are left. The second $\frac{1}{4}$-pound leaves $\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$, so the third $\frac{1}{4}$-pound bag leaves no sand.

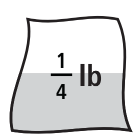


Writing this as a division problem, $\frac{3}{4} \div \frac{1}{4} = 3$. At first, it might be surprising that when dividing two fractions, the answer is an integer, especially when the integer is large compared to the fractions. What does 3 represent in this case?

Show on the number line why $\frac{3}{4} \div \frac{1}{4} = 3$.



What if the initial quantity is less than the bag size, like having $\frac{1}{8}$ pound of sand and a bag that holds $\frac{1}{4}$ of a pound. What is $\frac{1}{8} \div \frac{1}{4}$?



We return to the division $\frac{1}{8} \div \frac{1}{4} = \frac{\frac{1}{8}}{\frac{1}{4}}$ expressed as a fraction. If we multiply the numerator and denominator by the reciprocal of $\frac{1}{4}$, namely by 4, then:

$$\frac{1}{8} \div \frac{1}{4} = \frac{\frac{1}{8}}{\frac{1}{4}} = \frac{\frac{1}{8}}{\frac{1}{4}} \cdot \frac{\frac{4}{1}}{\frac{4}{1}} = \frac{\frac{4}{8}}{\frac{4}{4}} = \frac{\frac{4}{8}}{1} = \frac{4}{8} = \frac{1}{2}$$

In general, when the denominator of a fraction is a fraction, multiplying both the numerator and denominator by the reciprocal of the denominator produces a simpler fraction.

Another approach to simplify complicated fractions uses the pattern that $m \div n = \frac{m}{n} = m \cdot \frac{1}{n}$. Using this pattern, rewrite $\frac{\frac{1}{8}}{\frac{1}{4}}$ as $\frac{1}{8} \cdot \frac{4}{1}$, because the reciprocal of $\frac{1}{4}$ is $\frac{4}{1}$. Then multiply to find the answer: $\frac{1}{8} \cdot \frac{4}{1} = \frac{4}{8} = \frac{1}{2}$.

PROBLEM 3

Compute the following division of fractions using the stacking method from the previous page.

a. $2 \div \frac{1}{4}$

b. $3 \div \frac{1}{4}$

c. $\frac{1}{2} \div \frac{1}{4}$

d. $\frac{1}{4} \div \frac{1}{2}$

PROBLEM 4

Valerie's bird feeder holds $\frac{5}{6}$ of a cup of birdseed. Valerie is filling the bird feeder with a scoop that holds $\frac{1}{6}$ of a cup. How many scoops of birdseed will Valerie put into the feeder? Use the numerical technique from above. Write your answer in simplest form.

To summarize: We have two techniques for dividing or simplifying fractions:

Method 1: Write the division problem as a fraction and multiply the numerator and denominator of this fraction by the reciprocal of the denominator. This results in an equivalent fraction with denominator 1:

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{\frac{a}{b} \cdot \frac{d}{c}}{\frac{c}{d} \cdot \frac{d}{c}} = \frac{\frac{a}{b} \cdot \frac{d}{c}}{1} = \frac{a}{b} \cdot \frac{d}{c}$$

Method 2: Or, because division is equivalent to multiplication by the reciprocal, rewrite the division as multiplication:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

SUMMARY (What I learned in this section)

RATES, RATIOS, AND PROPORTIONS 7

Name:_______________________________ **Date:**_______ **Period:**______

SECTION 7.3 Rates and Ratios

VOCABULARY

DEFINITION	EXAMPLE
Ratio:	
Rate:	
Unit Rate:	

Big Idea: How do you use rates and ratios to solve problems?

EXPLORATION 1: WRITING RATIOS

Fractions are often used to compare quantities. For example, Miller Middle School has 400 students, and 280 of them live within 2 miles of the campus. Simplifying, $\frac{280\ students}{400\ students} = \frac{7}{10}$. Notice that both units are "students" and the fraction simplifies. How can you interpret the meaning of the simplified fraction?

____

In Miller Middle School, 7 out of every 10 students, or 70% of the students, live within 2 miles of the school. The fractional form of this comparison is called a ratio. A **ratio** is a division comparison, $\frac{7}{10}$, of two quantities with or without the same units.

There are three ways to write a ratio. Ratios can be written in the form of first one quantity, then a colon followed by a second quantity. Because there are 280 students who live within 2 miles, we would write (280 students who live within 2 miles) : (400 total students) or simplify 280 : 400.

Another way to write the ratio is with the word "to" in place of the colon. For example, 280 students *to* 400 students.

The third way we could express this ratio is by writing it in fraction form $\frac{280}{400}$. Ignoring for a moment the units and only using numbers, we would write 280 : 400 or 280 to 400 or $\frac{280}{400}$. Just as with fractions, ratios can be simplified. On the lines below, rewrite the ratio 280 : 400 in simplest form using all three methods.

____________________ ____________________ ____________________

Always remember what kinds of things are being compared. For example, if there are 42 pink hats for every 50 blue hats, and we want to write a ratio that compares these two quantities, we could begin by writing a variable ratio of what we're comparing. We'll let *p* represent pink hats and *b* represent blue hats. The ratio we want to write is pink to blue or *p : b*. Now we can insert numbers:

$$p : b$$
$$42 : 50$$

This ratio, like a fraction, can simplify. If it helps, write the ratio in fraction form. It does not matter what way you write the ratio, all three ways can be expressed in simplest form. In the example above, 42 : 50 would simplify to 21 : 25; 21 to 25; or $\frac{21}{25}$.

Let's use the animals on the Lucky J Ranch to practice writing ratios. On the ranch there are 8 horses, 54 cows, 2 bulls, 6 rabbits, 9 ducks, 1 rooster, and 56 chickens. Write each ratio in simplest form using the three different ways we learned.

a. horses to cows ____________________

b. ducks to chickens ____________________

c. rooster to chickens ____________________

d. bulls to cows ____________________

e. horses to rabbits ____________________

f. ducks to rabbits ____________________

g. rabbits to ducks ____________________

Look at problem f and g above. What do you notice about these two comparisons?

From your observation above, do you think that order matters when writing a ratio? Why or why not?

EXPLORATION 2: SPECIAL RATIOS

Rates are special ratios that compare different units. Suppose you earn 30 dollars for doing 5 hours of yard work and mowing the lawn. You know that 30 ÷ 5 = 6 indicates how much money you earned per hour. Using fractions, this calculation looks like $\frac{30}{5} = 6$. However, it is usually helpful to write this problem using the units that describe each quantity. So the calculation becomes

$$\frac{30\ dollars}{5\ hours} = \frac{6\ dollars}{1\ hour} = 6\ \frac{dollars}{hour}.$$

You read "6 $\frac{dollars}{hours}$" as "six dollars per hour." The answer explains exactly how many dollars you earned each hour. This quantity is an example of a rate. A **rate** is defined as a division comparison between two quantities, usually with two different units, like dollars and hours. What are some other rates that you have worked with or know about? The simplified fractional answer in the example is called a **unit rate** because it represents a number or quantity per 1 unit, or hour in this case. The units may be written in fractional form, like $\frac{dollars}{hours}$, $\frac{miles}{hour}$ or $\frac{miles}{gallon}$.

Let's try writing rates and unit rates.

Juan drove 150 miles in 3 hours and used 5 gallons of gasoline. Make as many rates using these quantities and their units as possible. Explain what each unit fraction means.

____

____

EXAMPLE 1

Wendy's Bakery uses 4 cups of flour per cake when making cakes. How many cups of flour will she use when she bakes 7 cakes for a customer?

We begin by writing a ratio to compare cups of flour to number of cakes: ____________________

Notice since we are comparing the number of cups to just *one* cake, we are writing a special ratio called a ________________________________.

We can use multiplication to find the number of cakes that 4 cups of flour will make.

$$4\ \frac{cups}{cake} \cdot (7\text{ cakes}) = 28\ \frac{cups}{cake} \cdot \text{cake} = 28\text{ cups}$$

Notice that the unit of "cake" in the numerator and denominator simplify to one and the answer is in cups. This is similar to computing the product $\left(\frac{7}{4}\right)(4) = 7$. This way of keeping track of the units is very useful in application problems, especially in science.

EXAMPLE 2

In each scenario below, write the unit rate as shown in the example using the word "per", meaning "each one".

Ex. 32 sheets of paper for every 2 pencils:

32 sheets: 2 pencils = 16 sheets per pencil

a. 120 km in 6 hours
____

b. 58 copies for 29 students
____

c. 63 books shared among 9 readers ____________________

d. 100 water games for 20 swimmers ____________________

e. 35 evening gowns and 7 models ____________________

EXPLORATION 3: TABLES AND RATIOS

Gloria works as a computer consultant for the Bayou Company and earns $612 for working 36 hours. If she charges a fixed amount per hour, how much will she earn working 18 hours? 9 hours? 1 hour? 4 hours?

Let's create a table to answer this question:

Hours	**36**	**18**	**9**	**1**	**4**
Earnings, ($)	**612**				

Notice as you go across the top row, **Hours**, that the value 36 is divided in half to arrive at 18. Therefore, you should perform the same operation in the **Earnings ($)** row. What is 612 ÷ 2? ____________ Write that in the appropriate cell of your table.

What happens to 18 in order to get to 9? As before, repeat that operation to find the next value in the **Earnings ($)** row. Continue until you find the amount for just one hour of work, also known as the ________ ________.

Now that you know the amount Kristen earns *per* hour, you can use that information to discover her earnings for any given hours she might work. Use the unit rate to help you find her earnings for four hours of work. Complete filling in the table.

Let's try a similar problem.

EXAMPLE 3

A space shuttle traveled 525 miles in only 50 seconds. If it traveled at a constant speed, how far did it travel in 10 seconds? 2 seconds? 1 second? 33 seconds? Use the table below to record your data:

Seconds	**50**	**10**	**2**	**1**	**33**
Distance, (mi.)	**525**				

Remember, sometimes our answers are whole numbers, and sometimes we must use a decimal or fraction. Show all work in the space below.

PROBLEMS

1. Diana sent 300 text messages in 5 hours. Find the number of messages Diana sent each hour.

   Diana sent ____________ messages per hour.

2. Norman rode his moped for 3 hours and traveled 123 miles. What was his average rate, or speed? ____________________

3. Referring to the problem above, approximately how far did Norman travel in the first hour and a half? ____________________

4. Adam and Lisa both jog every day. Adam jogs an average of 2,200 meters in 40 minutes and Lisa jogs an average of 1,500 meters in 30 minutes. Who jogs faster? ___________

5. Referring to the previous problem, on average, how far does Adam jog in 12 minutes? ____________

6. Using the information in problem 4, how long does it take Lisa to jog 600 meters, if she maintains her pace? ____________

7. Savannah packed the picnic basket with the following: 4 sandwiches, 6 fruit cups, 2 big bags of chips, 3 bottles of water, 2 diet colas, 1 bottle of lemonade, and a dozen chocolate chip cookies. Using the lines below, first write a variable ratio followed by a numeric ratio in simplest form. In your answers, use the three different forms for writing a ratio.

   a. water to cookies ____________
   b. all drinks to sandwiches ____________
   c. lemonade to number of cookies ____________
   d. all food items to all drink items ____________
   e. chips to diet colas ____________
   f. fruit cups to chips and sandwiches ____________
   g. sandwiches and chips to all items ____________

SUMMARY (What I learned in this section)

RATES, RATIOS, AND PROPORTIONS 7

Name:________________________________ **Date:**_______ **Period:**______

SECTION 7.4 Proportions

VOCABULARY

DEFINITION	EXAMPLE
Proportion:	

Big Idea: How do you use proportions to solve problems?

EXPLORATION 1: SETTING UP AND SOLVING PROPORTIONS

When you look at a map of Texas, you know that the actual state is much larger than the map. For example, according to a scale designation on the map legend, 1 inch can represent 50 miles. That means that the ratio of the map distance to the actual distance is 1 inch to 50 miles. This ratio is written 1 inch: 50 miles or $\frac{1\ in}{50\ mi}$, as in Sections 7.3.

Using this information, what actual distance does 2 inches represent?

Let's begin by writing our units ratio followed by the ratio, 1 inch to 50 miles, as shown below:

$$\frac{\textit{inches}}{\textit{miles}} = \frac{1 \text{ inch}}{50 \text{ miles}}$$

Since we know the inch to mile ratio is constant, we will set it equal to the ratio of 2 inches to *x*, or unknown, miles.

$$\frac{\textit{inches}}{\textit{miles}} = \frac{1 \text{ inch}}{50 \text{ miles}} = \frac{2 \text{ inches}}{x \text{ miles}}$$

In a proportion, each side of the equation is a ratio. Sometimes, a proportion can compare two different types of units such as miles to inches or the same units, like inches to inches and miles to miles, as long as both ratios are equivalent as fractions: $\frac{x\ mi}{2\ in} = \frac{50\ mi}{1\ in}$ or $\frac{2\ in}{1\ in} = \frac{x\ mi}{50\ mi}$

Let's look at three different methods for solving the same problem:

EXAMPLE 1

A colony of leafcutter ants cuts up 4 leaves in 7 minutes. Write the proportion that corresponds to this relationship. How many leaves does the colony cut in 35 minutes?

You can construct a table to record the time and the number of leaves cut. Finish the table below.

Time in Minutes	Number of Leaves Cut
0	0
7	4
14	

From the table you can see that 35 minutes corresponds to ______ leaves cut by the leafcutter ant.

Unit Rate Method

Set up a proportion that compares the ratio of leaves to minutes. Because the ants cut 4 leaves in 7 minutes, using division, the ants must cut $\frac{4}{7}$ of a leaf in 1 minute. This is the unit rate or the number of leaves cut per minute. If the ants keep cutting at this rate, they will cut 35 times this number of leaves in 35 minutes. Call the number of leaves cut in 35 minutes *x*. Then

$$x = \frac{4\ leaves}{7\ minutes} \cdot 35 \text{ min} = \frac{140}{7} \text{ leaves} = \_\_\_\_\_\_\_\_$$

Proportion Method

Set up a proportion by comparing amounts for the two different times.

The ants cut 4 leaves in 7 minutes. How many leaves *L* will the ants cut in 35 minutes?

$$\frac{L\ leaves}{35\ minutes} = \frac{4\ leaves}{7\ minutes}$$

What variable is being used in the proportion above? ______________

To solve, multiply both sides of the equation by the denominator 35.

$$35 \text{ min} \cdot \frac{L\ leaves}{35\ minutes} = \frac{4\ leaves}{7\ minutes} \cdot 35 \text{ min}$$

Let's look at the left side of the equation. To the left of the equality sign we have 35 min · L ÷ 35 min. What happens when you multiply a number times 35 and then divide it by 35?

____

This proportion method involves the rate of change in the form of speed, the rate of leaves cut per unit time or minute. This is a rate of change like miles per hour or mph.

EXAMPLE 2

Set up the following proportion and solve:

3 bags of chips cost $2.79. How much do 7 bags of chips cost?

We begin by writing a ratio to compare 3 bags to the cost in dollars: ___________________

Next, use a variable to write a ratio for 7 bags and the unknown cost: _______________

Write these two ratios as a proportion by setting them equal to each other and solve for the cost of 7 bags of chips:

Do this problem using unit rates. Which method do you prefer?

PROBLEM

Write the ratios in fraction form. Can you write the ratios as proportions? Show how? If the ratios form a proportion, write an = symbol between them. If not, write the ≠ (not equal) symbol.

2:7 and 30:126	24:8 and 6:2	6:20 and 3:10
9:6 and 6:2	3:27 and 5:125	5:11 and 11:5
5:27 and 30:162	4:7 and 60:105	51:17 and 3:1

EXAMPLE 3

Find the unknown value, x, in the following proportion:

$$\frac{7}{15} = \frac{x}{9}$$

One way is to multiply both sides by 9 then solve this equation as we see below.

$9 \cdot \frac{7}{15} = 9 \cdot \frac{x}{9}$

$\frac{63}{15} = \frac{9x}{9}$

$\frac{21}{5} = x$

Try the process on your own. Write the ratios in fraction form. Write an equation and solve using the tabular method, unit ratio method, or proportion method. Round your answers to the nearest tenth, if necessary.

$6 : 14 = x : 20$	$1 : 9 = x : 43$	$x : 11 = 7 : 17$
$x : 16 = 14 : 5$	$1.5 : 8 = x : 2.2$	$x : 21 = 16 : 31$

EXPLORATION 3: PROPORTION APPLICATION

You will need a map of any region that contains a legend with the distance scale and a ruler or tape measure in the same unit system as the map.

Step 1: Find the legend in the map and write a ratio that relates the map measure to the actual measure.

____

Step 2: Use a measuring instrument to measure the straight-line distance between two major cities on the map. ____________________________

Step 3: Determine the actual straight-line distance between the cities using proportions.

____

Step 4: Repeat Steps 2 and 3 with two other cities. ____________________________

What are the actual straight-line distances between the cities that you chose?

____

____

EXPLORATION 4: CURRENCY CONVERTER

The currency in the United States is in dollars and cents. Do you know the currency of our neighbors Mexico? Canada? What is the currency in England? France? Japan? China?

1. Find the currency for six different countries. You may need to use a reference or some outside source.
2. Determine what $1 US is worth in the currency of each of the six countries. You may need to use the internet for the most current exchange rate.
3. Determine what $50 is worth in each of the six currencies.
4. Using the currency of one of your countries, determine what 100 of that currency is worth in dollars.

Let's create a table to organize our data:

List 6 other countries of your choice and the name of the currency used in each. Then, calculate how much of the other currency equates with $1, and $50.

Country & Currency	**Currency equivalent to $1**	**Currency equivalent to $50**
United States/ dollar	$1	$50

Now, pick one of the other countries to answer question 4, and write your answer below.

100 ________________________ is worth ________________________ dollars.

EXPLORATION 5: INDIRECT MEASURES

Sometimes the measurement that we want to know is not easily accessible. For example, you see a tall building that has a fifteen feet shadow. You can measure the shadow because it is easy to measure on the ground, but measuring the building is a challenge you weren't prepared to take on. You see a small tree that is only 12 feet tall and has a 2 feet shadow. You can use a proportion to find the measure that you want. First, write the variable ratio for the object and the shadow. Next, write the ratio for the measurements you know, the tree and its shadow. Finally, set that ratio equal to the ratio of the unknown building height to its shadow. Use the space below to set up your proportion and solve to find the height of the building.

The building is _______ ft. tall.

PROBLEMS

1. Alberta likes to knit socks for her grandson's collection of toy aliens, but she has forgotten how many legs each alien has. She remembers knitting 24 socks for 3 aliens. Assuming that the aliens all have the same number of legs, how many socks should she knit for 5 aliens?

2. Norman collects model racecars. For every 3 red cars he collects, he buys 2 blue models. If he has 24 blue cars, how many red cars does he have?

   Write a units ratio followed by a proportion to solve: ____________________________

3. Reynaldo creates balloon bouquets at his flower shop using blue, silver, and white balloons. The ratio of blue to silver balloons is 5:2 and the ratio of blue to white balloons is 5:1. If a bouquet contains 11 white balloons:

   a. How many blue balloons does each bouquet contain? _____________________

   b. How many silver balloons does each bouquet contain? _____________________

   c. How many balloons does each bouquet contain altogether? _____________________

4. Zach's car drives 22 miles on one gallon of gas. How many gallons will he need in order to travel: (Set up a proportion to solve.)

   a. 50 miles?_______________

   b. 100 miles? ______________

   c. 2 miles? ________________

5. Alex can type 39 words per minute. If he always types at the same rate, how long will it take him to type a 500-word report? Use a proportion to solve. Round your answer to the nearest tenth, if necessary.

   It will take Alex _____________ minutes.

6. It costs $128 for every 4 tickets purchased at the water park. If this rate is constant, how much will 6 tickets cost? Use a proportion to solve.

   It will cost __________________.

7. A 6-ft. tall man casts a 15-ft. tall shadow. He is standing next to a tree that casts a 35-ft. shadow. Use a proportion to find the height of the tree.

   The height of the tree is __________________.

SUMMARY (What I learned in this section)

RATES, RATIOS, AND PROPORTIONS 7

Name:____________________________ Date:________ Period:_______

CHAPTER 7: SPIRAL REVIEW

1. Write the steps used to multiply fractions and show an example.

   ____

   ____

   ____

   ____

2. Carlos is 5 feet tall and casts a 12-feet shadow, while a tree in his yard casts a 20-feet shadow. Use a proportion to find the height of the tree.

   The tree is ___________ feet tall.

3. Blake buys a gym bag for $33 and three equally priced athletic shorts. The total is $96. What is the price of each shirt?

Each shirt costs ________________.

4. Three packs of sour candy costs $1.89. Write a proportion and solve to find the cost of 10 packs of candy.

10 packs of candy cost ________________.

5. Tim can run a lap around the track every 3 minutes, while Tom can run a lap every 4 minutes. If they start together, how many minutes will pass before they are crossing the starting line together again. Assume they run at a constant speed.

Tim and Tom will cross the starting line together again in ________ minutes

6. Evan is riding his bike at a speed of 7 km per hour. How far will he travel in two and a half hours?

In $2\frac{1}{2}$ hours, Evan will travel __________ km.

7. Evaluate the following expressions:

a. $36 \div 3^2 \cdot 12$

____

____

____

____

b. $5^2 + 20 \cdot (2 + 4)$

____

____

____

____

8. Consider the sequence below:

   1, 6, 11, 16, 21, …

   The next two terms in the sequence would be: ___________, __________

   Write the rule to find the nth term: ____

9. Solve the following equations:

$6p = 73.2$	$86.5 + x = 14$	$54.7 + t = 63.09$
$4.96 + k = 8.2$	$v - 36.5 = 12.4$	$15.5 \div f = 3.1$

10. There are 16 ounces in a pound. If a newborn baby weighs 7.5 pounds, what is its weight in ounces? Write a proportion to solve.

The baby weighs __________ oz.

MEASUREMENT 8

Name:________________________________ **Date:**_______ **Period:**______

SECTION 8.1 Length

VOCABULARY

DEFINITION	EXAMPLE
Customary Units:	
Metric Units:	
Dimension Analysis:	
Meter:	

Big Idea: How do you measure length? What units are used to measure distances and lengths?

What do you notice common to the following questions?

- How far is your home from school?
- How tall are you?
- What is the record temperature in San Marcos for today?
- How much water do you consume in one day?
- How much did you weigh when you were born?
- How much time until lunchtime?

You probably observed that they involve measurements of some type.

Whether it is measuring distance, height, temperature, capacity, weight, or time there are several important concepts to keep in mind. First, you must determine what is being measured. For example, if you want to know how tall you were when you were born, you would recognize that you are referring to measuring a length and not weight, capacity, or time. Second, you must determine what unit or possibly units are appropriate for measuring the height of a baby. And finally, you must get the actual numerical value for the measurement in the unit chosen.

Throughout this chapter, we will look at two systems of measurement: **customary units** and **metric units**. Customary units are what we in the U.S. use most often though the rest of the world, with a few exceptions, uses metric units.

EXPLORATION 1: CUSTOMARY UNITS

What are customary units you use for lengths and distances? How do you measure length?

Use an appropriate measuring devise such as a ruler, meter stick, measuring tape, or other available device to measure five different objects using five different units. Explain why you chose the particular unit.

Record your findings and responses in the table below.

Object Measured	Measurement	Why did you choose this unit?
1.		
2.		
3.		
4.		
5.		

Inches may be a good choice for the height of a newborn baby but not as good a choice for a running race distance. Here are some useful conversions in customary units:

LENGTH Customary
1 mile = 1760 yards
1 mile = 5280 feet
1 yard = 3 feet
1 foot = 12 inches

We introduce a very powerful process for converting units called **dimensional analysis**. This process is based on equivalent forms of measurement. Recall that a fraction of the form $\frac{n}{n} = 1$, with n a non-zero number. You would easily recognize that $\frac{15}{15} = 1$. Now consider $\frac{1\text{ foot}}{12\text{ inches}}$. Because 1 foot = 12 inches, then a non-zero quantity divided by the same non-zero quantity is equal to 1. What is important here is that the unit must be indicated every time. Remember, we are not claiming that in general, $\frac{1}{12} = 1$. Units are important!

Create other 1's using units with equivalent forms.

Some typical equivalents with the corresponding conversion rates equal to 1 are:

3 feet = 1 yard, which means $\frac{3\,ft}{1\,yd} = 1 = \frac{1\,yd}{3\,ft}$

AND

60 minutes = 1 hour, which means $\frac{60\,minutes}{1\,hour} = 1 = \frac{1\,hour}{60\,minutes}$

Rewriting a measurement from one unit to another is much easier using dimensional analysis.

EXAMPLE 1

Convert 31,680 inches to feet. Recall that 1 foot = 12 inches.

EXAMPLE 2

Convert 2,640 feet to miles (recall that 1 mile = 5,280 feet).

EXPLORATION 2: METRIC UNITS

Another system for measuring length is the metric system. This system is a base 10 or decimal system. Your knowledge of decimals will be very useful in the metric system. The base unit of length in the metric system is the **meter**. Prefixes are used with the base to create larger and smaller units.

Prefixes commonly used in order from larger units to smaller units are:

Metric unit Prefixes

Kilo	Hecto	Deka	(base)	Deci	Centi	Milli
k	h	da		d	c	m

The unit to the left of a given unit is 10 times larger than the one to its right.

EXAMPLE 3

Henry measured the length of his patio and found it to be 1,200 cm long. He went to purchase an outdoor rug to cover the length of the patio. He noticed it was sold in meters. How long is the patio in meters?

PROBLEMS

1. Convert the following customary measurements to the indicated units. Make sure to show your work and set up equivalent units correctly.

a. 24 inches = ______ ft	b. 12 ft = _______ inches	c. 3 miles = _______ ft
d. 136 inches = _______ yards	e. 7,040 yd = _______ miles	f. 18 yd = _______ ft

2. Convert the following metric measurements to the indicated units. Make sure to show your work and set up equivalent units correctly.

a. 3,250 cm = ______ m	b. 4,000 m = _______ cm	c. 140,000 mm = _______ km
d. 32 mm = _______ m	e. 5 km = _______ cm	f. 5,000,000 mm = _______ km

3. Convert 36 feet to the specified units. Show your work.

a. inches 36 feet = ______ inches.	b. yards 36 feet = ______ yards.	c. miles 36 feet = ______ miles.

4. Convert 50 meters to the specified units. Show your work.

a. millimeters 50 meters = ______ mm.	b. centimeters 50 meters = ______ cm.	c. decimeters 50 meters = ______ dm.

5. Darrell is 6 feet, 9 inches tall. How tall is he in just inches?

6. Mrs. McLean walked $2\frac{1}{2}$ miles on Sunday, $3\frac{3}{4}$ miles on Monday, and $1\frac{3}{4}$ mile on Tuesday. How many yards did she walk?

7. The city library is 2,795 meters from Miller Middle School. How many kilometers is the library from the middle school?

8. Find the dimensions (length and width) of this workbook. Record it below in inches and centimeters. Then convert the metric measurements to millimeters.

	Length of Workbook	Width of Workbook
Inches		
Centimeters	________ cm = ________ mm	________ cm = ________ mm

9. Convert 95,000 mm to kilometers. Show your work.

SUMMARY (What I learned in this section)

MEASUREMENT 8

Name:________________________________ **Date:**_______ **Period:**______

SECTION 8.2 Capacity and Volume

VOCABULARY

DEFINITION	EXAMPLE
Volume:	
Capacity:	
Liters:	

Big Idea: How do you measure volume? What units are useful for measuring volume and capacity?

The measure of the space in a container is called the **volume** of the container. We will study volumes of familiar shapes such as cubes more carefully in Chapter 9. You will see that cubic units are used to measure volume. In this section, we will explore the capacity of a container. **Capacity** refers to how much liquid a container can hold. Our examples are often in liquid form because it will easily conform to any shaped container. The customary units for capacity in decreasing order are gallons, quarts, pints, cups, and fluid ounces.

EXPLORATION 1: CUSTOMARY UNITS FOR CAPACITY

Let's examine some items in our classroom to determine which of the following customary units would be most useful for measuring their capacity: Gallons, quarts, pints, cups and fluid ounces.

Explain why you chose those units.

Predict for the capacities of each item that you found using two different units. Use a measuring cup and water to record and confirm your predictions.

Record your predictions and measurements in the table below.

Item	Capacity Prediction	Actual Capacity
1.		
2.		
3.		

We summarize the relationships among the customary units as follows:

CAPACITY Customary
1 gallon = 4 quarts (qt.)
1 quart = 2 pints (pt.)
1 pint = 2 cups
1 cup = 8 fluid ounces (fl.oz.)

EXAMPLE 1

Isabel is making soup that requires 3 quarts of water. She only has a 1-cup measuring cup. How many cups of water will Isabel need in order to make this soup?

EXAMPLE 2

Determine what fractional part

a. 1 quart is to 1 gallon

b. 1 pint is to 1 gallon

EXPLORATION 2: METRIC UNITS FOR CAPACITY

The metric system uses **liters** as the base unit for capacity. Just as with length measures, the prefixes will create larger or smaller units based on liters.

Write three other units that are larger than liters and three other units that are smaller.

The units with liter (L) as base are:

Metric unit for Capacity

Kiloliter	Hectoliter	Dekaliter	Liter	Deciliter	Centiliter	Milliliter
kL	hL	daL	(base) L	dL	cL	mL

The units that are used most often are the liter and milliliter. Notice that the relationship between the two units is 1 liter = 1000 milliliters, which can also be written as, 1 milliliter = 1 liter.

EXAMPLE 3

A camel drinks 20 liters of water a day. How many milliliters does this equal?

PROBLEMS

1. Convert the following customary measurements to the specified units. Make sure to show your work and set up equivalent units correctly.

a. 100 fluid oz. = ______ quarts.	b. 9 quarts = _______ pints	c. 20 fluid oz. = ______ cups
d. 18 pints = ______ fluid oz.	e. 6 gallons = _____ fluid oz.	f. 13 pints = _____ quarts

2. Convert the following metric measurements to the specified units. Make sure to show your work and set up equivalent units correctly.

a. 350 mL = ______ liters	b. 4 kL = _______ liters	c. 2,000 mL = _______ liters
d. 37 liters = ________ mL	e. 6 kL = ________ mL	f. 8,500,000 ml = _______ kL

3. Convert 120 fluid ounces to the specified units. Show your work.

a. gallons 120 fl. oz. = ______ gallons	b. quarts 120 fl. oz. = ______ quarts	c. pints 120 fl. oz. = ______ pints

4. Convert 9,320 milliliters to the specified units. Show your work.

a. liters 9,320 mL = ______ liters	b. kiloliters 9,320 mL = ______ kiloliters	c. centiliters 9,320 mL = ______ centiliters

5. Amy measured the amount of cooking oil in her deep fryer and found that it contained 4,700 mL. How many liters of cooking oil was in the fryer?

6. Sophia's aquarium holds 40-gallons of water. After cleaning the aquarium, she finds that she must add $32\frac{1}{2}$ quarts of water to fill the tank to capacity. How much was in the aquarium before she added the water?

7. Patrick's rain gauge measured 50 centiliters of rain in March, 1,267 milliliters of rain in April, and 12 deciliters of rain in May. How many liters of rain was collected during the three month period? Which month had the most rainfall?

8. After completing a 5K, April drinks 6 pints of water, May drinks 18 cups of water, and June drinks 1 gallon of water. Who drank the most water? Be sure to show your work to prove your answer!

9. After painting his office, Fineas had 18 pints of paint left over. If he began with 6 gallons of paint, how much paint did he use?

10. Determine the fractional parts:

   a. 1 cup is to 1 gallon ________________

   b. 1 milliliter is to 1 liter ________________

   c. 1 liter is to 1 kiloliter ________________

   d. 1 ounce is to 1 pint ________________

   e. 1 cup is to 1 quart ________________

SUMMARY (What I learned in this section)

MEASUREMENT 8

Name:________________________________ **Date:**_______ **Period:**______

SECTION 8.3 Weight and Mass

VOCABULARY

DEFINITION	EXAMPLE
Weight:	
Mass:	
Tons:	
Pounds:	
Ounces:	
Grams:	

Big Idea: How do you measure weight? What units are useful for measuring weight?

How heavy is an object? This question refers to the **weight** of an object. We often refer to the **mass** of an object in terms of weight but we should note that there is a scientific distinction.

The customary units for weight in decreasing order are **tons**, **pounds**, and **ounces**.

EXPLORATION 1: CUSTOMARY UNITS FOR WEIGHT AND MASS

Examine the items that you brought from home and determine the mass and weight of each item. Feel how heavy each item weighs. Find two items in the classroom and make an educated guess about their weights.
Here are some useful relationships among the customary:

MASS AND WEIGHT Customary
1 ton = 2000 pounds (lbs.)
1 pound = 16 ounces (avoirdupois) (oz.)

EXAMPLE 1

A female African elephant weighs approximately 7,900 pounds. Convert the weight to tons.

Now, convert 7,900 pounds to ounces.

EXPLORATION 2: METRIC UNITS FOR WEIGHT AND MASS

The metric system uses **grams** as its base unit for weight measure. The abbreviation for grams is simply the letter *g*. The prefixes will give us larger and smaller units. What units would be larger weight measures? What units would be the smaller weight measures?

In the table below are the metric units for weight with gram as base:

Metric units for Weight

Kilogram	Hectogram	Dekagram	Gram	Decigram	Centigram	Milligram
kg	hg	Dg	(base) g	dg	cg	mg

Do you know what weighs 1 gram (g)? A cubic centimeter of water weighs 1 gram. A paper clip is approximately 1 gram. In other words, a gram weighs very little. 450 grams is approximately equal to one pound. 1 kilogram is approximately equal to 2.2 pounds.

EXAMPLE 2

A female Asian elephant weighs approximately 3,000 kilograms. Convert the weight to grams.

PROBLEMS

1. For the following objects, list the most appropriate units of measurement: ounces, pounds, tons, grams, or kilograms.

Object	Customary Unit	Metric Unit
Handful of grapes		
School bus		
Backpack full of books		

2. Convert the following customary measurements. Make sure to show your work and set up equivalent units correctly.

a. 4,500 lbs. = ______ tons	b. 40 oz. = _______ lb.	c. 7 tons = _______ oz.

3. Convert the following metric measurements. Make sure to show your work and set up equivalent units correctly.

a. 300 mg = ______ grams	b. 3 kg = _______ mg	c. 10 g = _______ dg

4. Yosh has 7 lbs. 5 oz. of hamburger meat. How many 9 oz. burgers can he make?

5. The local rock quarry produced 3,625 pounds of limestone, 7,800 pounds of granite, and 2,375 pounds of marble. How many tons of rocks were produced?

6. Penelope weighs 49.8 kilograms and Victoria weighs 52,632 grams. Who weighs more? How much more?

____________________ weighs ________________ more than __________________.

7. On a recent trip to the candy store, Parker bought 2.3 kg of jellybeans, Carson bought 175 grams red hots, and Hannah bought 970,000 mg of gummy worms.

   a. How many grams of candy did the three children purchase altogether?

   b. Who bought the most candy (by weight)?

   c. If Carson decided not to buy any candy, how many grams of candy would the children have altogether?

8. Convert 95,000 mm to kilometers. Show your work.

SUMMARY (What I learned in this section)

MEASUREMENT 8

Name:______________________________ Date:________ Period:_______

SECTION 8.4 Time and Temperature

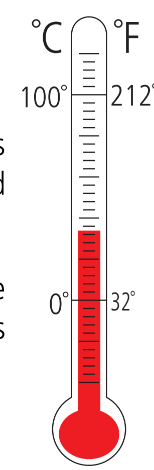
VOCABULARY

DEFINITION	EXAMPLE
Fahrenheit:	
Celsius:	

Big Idea: What units are used to measure temperature? time?

Does it make sense to say the temperature on a hot day in Texas is 32 degrees? Does 100 degrees make sense? Actually, the answer could be yes or no. How can that be? As you have seen in the previous sections, whenever you measure you must be careful to always include the units along with the numerical value.

EXPLORATION 1: COMMON UNITS OF TEMPERATURE



The two common units of temperature measure are the Fahrenheit and Celsius units. Fahrenheit is associated with the customary measure while Celsius is associated with the metric system.

You did an investigation in Section 1.1 regarding the thermometer as part of a number line. Let us recall some important aspects of the two units. The freezing point of water is 32° Fahrenheit and 0° Celsius. The boiling point of water is 212° Fahrenheit and 100° Celsius.

Now 32° in Fahrenheit is not a reasonable temperature on a hot day in Texas. Look on the thermometer to the right. Does 32° Celsius make sense for a hot day? 32° Celsius is approximately equal to 90° Fahrenheit, which is a reasonable temperature for a Texas summer.

EXAMPLE 1

What is a reasonable springtime temperature in Texas? Write your answers in both Fahrenheit and Celsius ________________

What is the average body temperature of a healthy person? Write your answers in both Fahrenheit and Celsius _____________

EXPLORATION 2: TIME

What units of time are most familiar to you? What units of time would be appropriate for each of the following instances?

a. What was the runner's time in a 100 meter run?
b. How long is your summer vacation?
c. How much time until dinner?
d. How much time will pass before you turn 21 years old?

We do not specify a customary or metric system of time measurement. However, some countries use a 24-hour reading of time while other countries use a 12-hour reading and use a.m. and p.m. to distinguish the morning time from the afternoon time. Generally, a.m. time goes from 12:00 midnight until 11:59 in the morning and p.m. goes from 12:00 noon until 11:59 at night. We will use the 12-hour clock in this book.

Familiar time equivalences include:

Time
1 year = 365 days
1 year = 12 months
1 year = 52 weeks
1 week = 7 days
1 hour = 60 minutes
1 minute = 60 seconds

EXAMPLE 2

12 hours and 20 minutes is equal to how many minutes? _______________

100 minutes is equal to how many hours? _______________

PROBLEMS

1. What fraction is:

   a. One day of a week? __________ c. One hour of a day? __________

   b. One day of a year? __________ d. One second an hour? __________

2. What fraction of an hour is:

   a. 20 minutes? __________ b. 15 minutes? __________

   c. 5 minutes? __________

3. List a reasonable temperature in Fahrenheit and Celsius (the two temperatures do not have to be equivalent) for each item below.

   a. What is a reasonable temperature for a hot day in El Paso? ______ F, ______ C

   b. What is a cold winter day in Chicago? ______ F, ______ C

4. The temperature in Anchorage, Alaska was -6° F at 6 a.m. and rose to 26° F by noon. How many degrees did the temperature rise?

5. Ms. Lott's begins math class at 11:47 a.m. The math classes are 50 minutes long. At what time will Ms. Lott's math class end?

6. Ms. Black's flight left Austin, Texas at 11:28 a.m. and arrived in Chicago, Illinois at 3:05 p.m. How long was her flight?

7. How many hours are in one week? How many minutes are in one week?

8. If Ms. Blackwell works 30 hours per week. How many hours will she work in one year?

SUMMARY (What I learned in this section)

MEASUREMENT 8

Name:____________________________ **Date:**_______ **Period:**______

CHAPTER 8: SPIRAL REVIEW

1. Use proportions to convert the following measurements:

   a. 104 in. = _________ ft.   b. 68 oz. = ____________ lbs.

2. Adam's mailbox is 3.5 ft. high and casts a 2 ft. long shadow. If Adam is 6 ft. tall, how long would his shadow be? Use a proportion to solve.

   Adam's shadow is ___________ feet tall.

3. Blake buys a gym bag for $33 and three equally priced athletic shorts. The total is $96. What is the price of each shirt?

   Each shirt costs _________________.

4. Lydia arrived home from school at 4:15 pm. If she was on the bus for 55 minutes, at what time did she board the bus?

Lydia boarded the bus at ________________________.

5. Emily is making flower arrangements as table centerpieces for a banquet. She has 32 roses, 24 lilies, and 56 daisies. What is the largest number of identical flower arrangements Emily can make?

Emily can make _______ flower arrangements. Each arrangement will have ______ roses, _______ lilies, and ______ daisies.

6. Eva can lift 127 pounds. What is this amount in?

Eva can lift ________ oz.

7. Write +, – , · , or ÷ in the blanks to make the following expressions true:

a. 6 ______ 3 _______ 6 = 24

b. 3 ______ 4 _______ 2 = 6

c. 6 ______ 5 _______ 3 = 8

d. 7 ______ 54 _____ 9 = 13

8. Sandra walked 17 yards while Missy walked 55 feet. Who walked farther?

________________________ walked farther.

9. Melinda left San Antonio, TX, at 4:03 pm and arrived in Fayetteville, AR, that same day at 7:42 pm. How long was her flight?

Melinda's flight was _______________________________ long.

10. Round each number to the place value indicated in the table.

Number	Ones (whole #)	Tenths	Hundredths	Thousandths
16.9257				
32.0198				
2.99542				
8.3977				
0.19191				
10.9428				
0.69842				
100.5873				
65.0296				
15.5658				

GEOMETRY 9

Name:______________________________ **Date:**________ **Period:**_______

SECTION 9.1 Measuring Angles

VOCABULARY

DEFINITION	EXAMPLE
Line:	
Line Segment:	
Ray:	
Angle:	
Straight Angle:	
Vertex:	
Acute Angle:	
Obtuse Angle:	
Right Angle:	
Protractor:	

Measure of Angle:	
Perpendicular:	
Supplementary:	
Complementary:	

Big Idea: How do we construct, measure, and classify angles?

EXPLORATION 1: LINES, LINE SEGMENTS AND RAYS

How would you answer the question, "What is an angle?"

You have probably seen angles in many places in everyday life. Can you name a few of these places?

In this section, you will learn what angles are, how to construct angles and how to measure the size of an angle.

Locate three points on the line below:

Label the left point *A*, the middle point *B*, and the right point *C*. Typically, two points on the line are used to identify a line. For example, if points *A* and *B* are used, then we use the notation, $\overleftrightarrow{AB}$. You can also use points *B* and *C*. Denote the line using two other points.

A line segment is a part of a line that includes two endpoints and all the points in between the points. For example, the line segment with end points A and B is written, $\overline{AB}$. Identify other line segments on this line that involve using a pair of the given points A, B, C.

A **ray** is also a part of a line that has a starting point and continues forever in only one direction. One ray on the line above is the ray that has starting point *B* and goes in the direction of *C*. We write $\overrightarrow{BC}$ for the ray *BC*. If we want to describe the ray starting at B that goes in the direction of *A*, what notation could you use? What other rays can you describe on the line above?

____

Notice that while line segments $\overline{AB}$ and $\overline{BA}$ describe the same line segment, the rays $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are very different parts of the line.

Let's now by construct an angle. To do this, first draw a point and label it point *P*. Next draw two points and label them *Q* and *R*. Connect points *Q* and *R* to point *P*, by drawing two separate rays.

We can now answer our original question: "What is an angle?"

Definition 9.1: Angle
An **angle** is formed when two rays share a common vertex.

The common endpoint *P* on both rays is called the **vertex** of the angle. In the diagram, the rays $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ form an angle called angle *QPR* or $\angle QPR$. The symbol "$\angle$" is the math symbol for the word *angle*. To name an angle, you can do the following three steps in order:

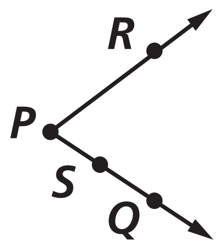
1. Write the name of one of the non-vertex points on one of the rays.
2. Write the name of the vertex.
3. Write the name of a non-vertex point on the other ray.

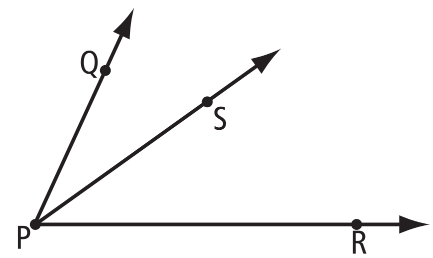
There can be many ways to name the same angle because there are many choices of points on the two rays in steps 1 and 3. You could also label this angle $\angle RPQ$. The order in which the points are written does not matter as long as the middle point identifies the vertex of the angle.

EXAMPLE 1

In angle $\angle XYZ$, identify the two rays that make up the angle and identify the vertex. It may help to draw the angle.

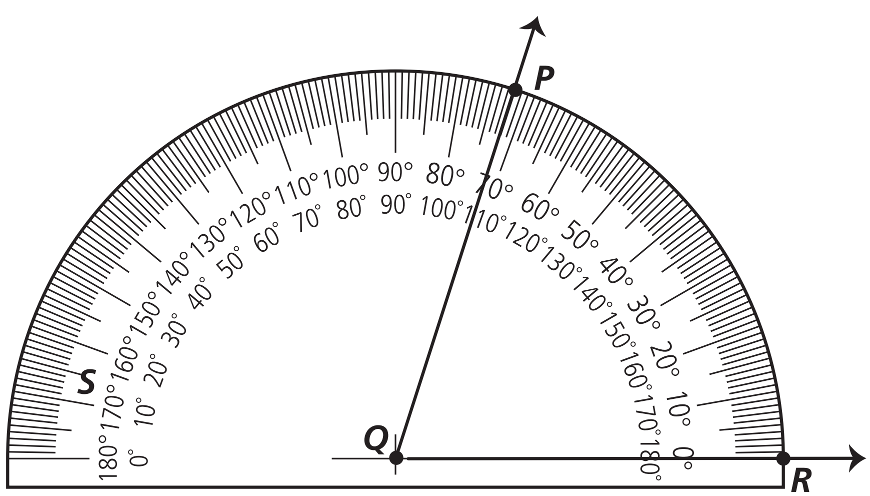
Sometimes a single letter is used to name an angle. Sometimes $\angle QPR$ is called angle *P*, or $\angle P$, when there is no confusion because there are only 2 rays beginning at P and therefore only one angle at P. Just the letter *P* is used to name the angle. For instance, in the left figure below, $\angle QPR$ is the same as $\angle P$. In the figure on the right, however, $\angle QPR$ is the largest angle but $\angle P$ could be one of three angles.





EXAMPLE 2

Once you understand the definition of an angle, the next step is to measure the size of the angle. __________________ are commonly used to measure angles. One degree is written ________ and is the angle formed by ___________ of a full revolution around a circle. An angle that makes a full revolution has a measure of _____________.



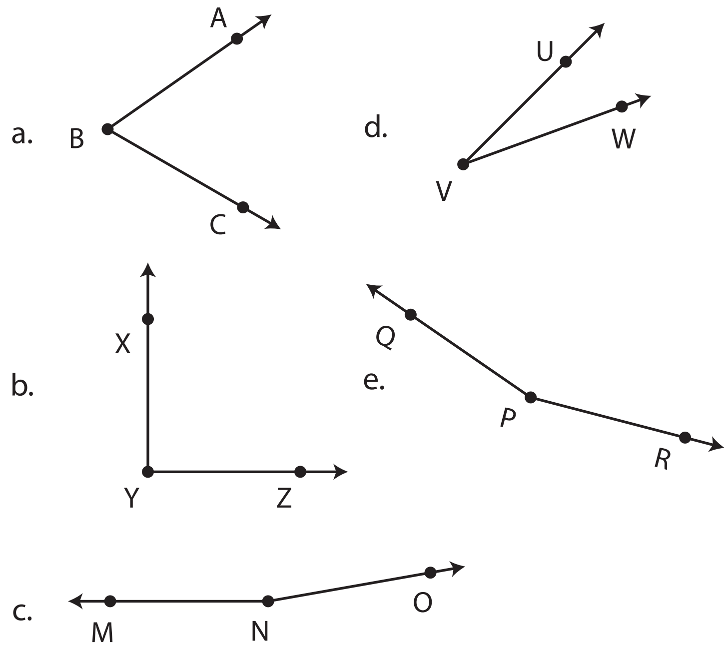
The instrument used to measure angles is called a _____________________. Protractors have degree markings along the outside of the curved edge. To measure an angle, place the vertex at the center of the semi-circle so that one ray passes through 0° or 180° and the other ray passes through a mark on the curved edge. If necessary, extend the other ray so that it falls on a mark along the curved edge. The degree at this mark is the measure of the angle or its supplement, which we will define later in this section.

If two rays with a common endpoint form a straight line, the angle they form has a measure of ________ degrees, or _________. This is called a ________________ angle. Angles that have a measure between ______ and _______ are called acute angles. Angles that have a measure greater than ________, but less than _______ are called obtuse angles. Angles that measure exactly _______ are called right angles.

Using a protractor, construct and label a straight angle, a right angle, an acute angle and an obtuse angle and indicate the measure of each angle.

EXAMPLE 3

Measure each of the angles below with your protractor:



2. Consider the rays and points above. Name each angle in two ways.

3. Classify each of the angles as acute, obtuse, right, or straight.

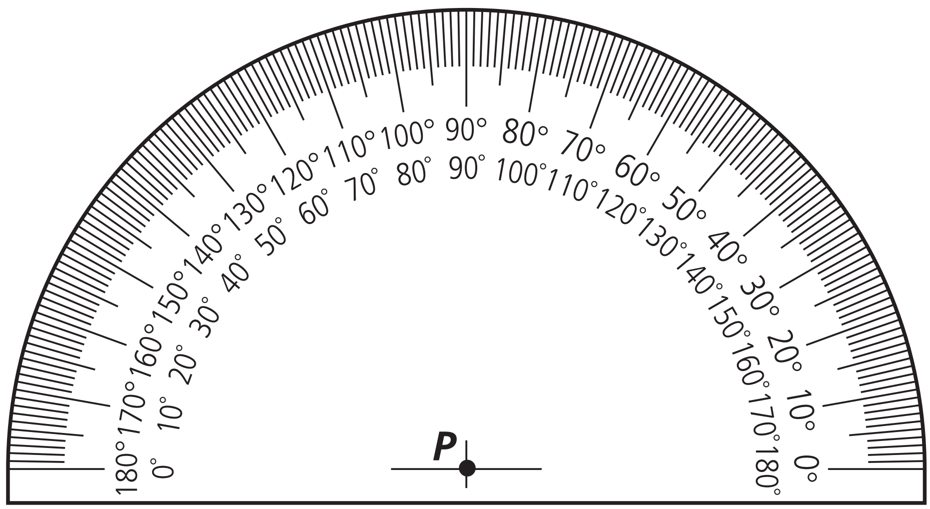
EXPLORATION 2: CONSTRUCTING ANGLES

1. Divide a straight angle into two parts. Describe how you constructed it to another student or your teacher. Measure each angle using your protractor.

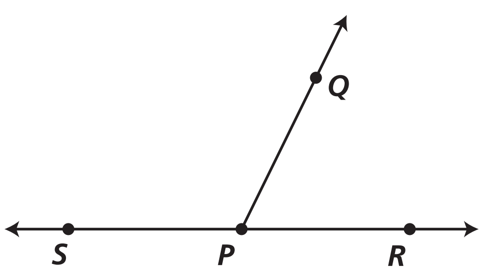
2. Use your protractor to draw rays making the following angles: 35°, 80° and 100°.

How do you construct an angle? Here is one approach to construct an angle with a given measure such as 64°.

1. Draw an initial ray and label $\overrightarrow{PR}$ it . The initial ray is usually, but not necessarily, horizontal.
2. Place the center of the semi-circle of the protractor on top of the point *P*, with the ray passing through 0°.
3. Find the place along the curved edge of the protractor that corresponds to the degree measure you are constructing and mark it with a new point *Q*.
4. Draw a line connecting the point *P*, which is the vertex of the angle, to the new point *Q* with a straight edge to obtain an angle of 64°.



EXPLORATION 3: MEASURE OF AN ANGLE



Mathematicians use the notation m(∠*QPR*) to mean the **measure of angle** *QPR*. Notice that ∠*QPR* and ∠*QPS* divide the ∠*RPS* into two parts. The measure of each angle is a number, and we can add these numbers together to get the equations below:

m(∠*QPR*) + m(∠*QPS*) = m(∠*RPS*)

SR is a line, so m(∠*RPS*) = 180°. Then we have

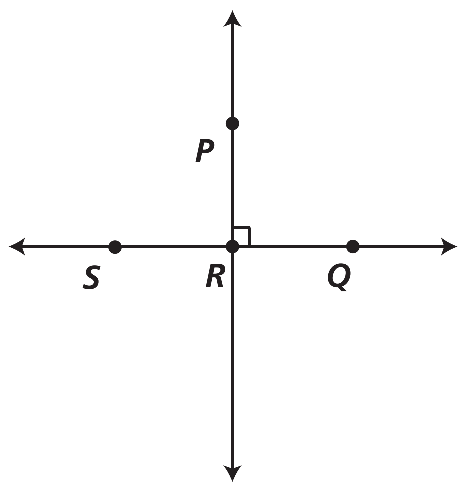
m(∠*QPR*) + m(∠*QPS*) = 180°.

Definition 9.2: Supplementary
Two angles are **supplementary** if the sum of their measures is 180°.

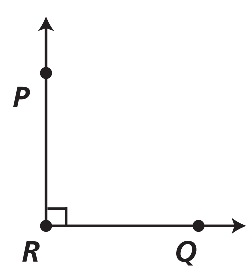
In the example above, $\angle QPR$ and $\angle QPS$ are supplementary angles. This means *$\angle QPS$ is the supplement of $\angle QPR$* and *$\angle QPR$ is the supplement of $\angle QPS$*.

Now divide a straight angle in half. Each angle formed is a right angle and measures 90° because $\frac{1}{2}$ of 180° is 90°. When two lines or line segments meet and form a right angle, they are **perpendicular** to each other.

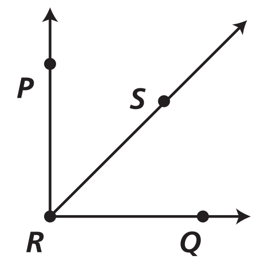
We often label the right angle with  to indicate that it is 90°.



When two rays meet to form a right angle, they are perpendicular rays.



Next, divide the right angle *PRQ* above into two parts:



Because $\angle PRS$ and $\angle SRQ$ divide $\angle PRQ$,

$$m(\angle PRS) + m(\angle SRQ) = m(\angle PRQ) \text{ and}$$
$$m(\angle PRS) + m(\angle SRQ) = 90°.$$

Definition 9.3: Complementary
Two angles are **complementary** if the sum of their measures totals 90°.

In the example above, $\angle PRS$ and $\angle SRQ$ are complementary angles. This means *$\angle SRQ$ is the complement of $\angle PRS$* and *$\angle PRS$ is the complement of $\angle SRQ$*.

EXAMPLE 4

Use the appropriate notation to write the measure of each angle described below:

a. right angle *XYZ*: ______________________________

b. angle *S*, the supplement of a 45° angle: ______________

c. angle *C*, the complement of a 23° angle: ______________

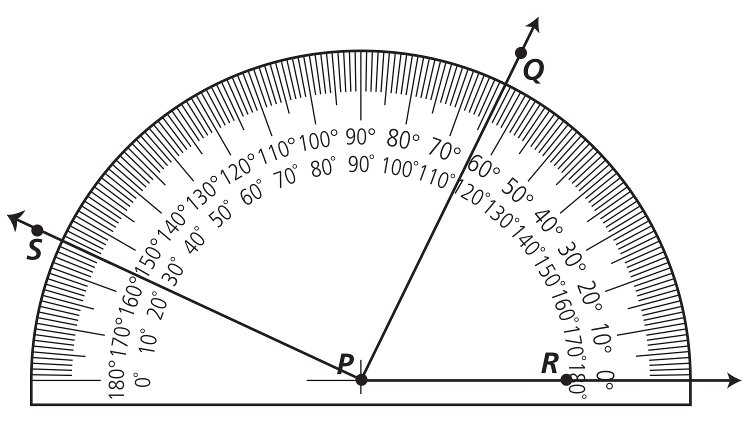
d. the sum of angle *C* and angle *S* above: ______________

PROBLEMS

1. Construct an angle starting at 0° with measure 45°.

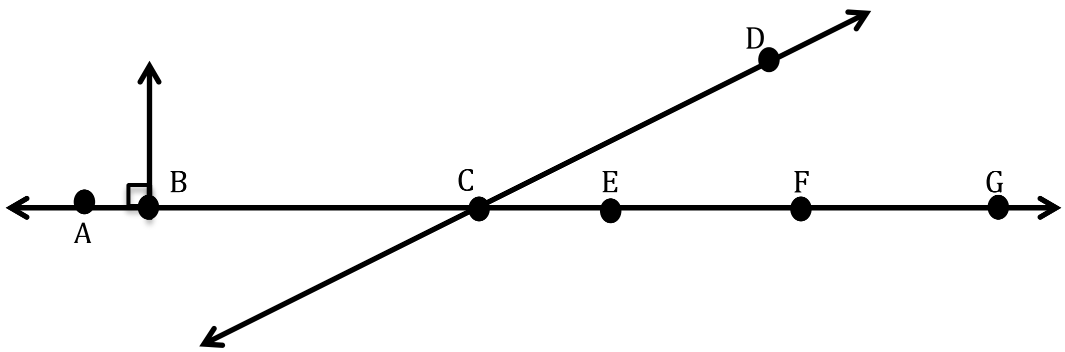
2. Construct an angle with one ray at 20° on the protractor with measure 45°.

3. Consider the rays on the protractor below. Measure the angles, QPS and SPR.



$m\angle QPS =$ ___________ $m\angle SPR =$ ________

4. Use the diagram below to answer the questions that follow. Write your responses using the appropriate notation.



a. Name 2 lines: ______________________________

b. Name 2 line segments: ______________________________

c. Name 2 angles that are supplementary: ____________________

d. Name a right angle: ______________________________

5. Use a protractor to measure the following angles using the figure in problem 4.

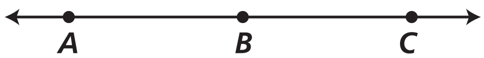
m(∠B) = ______________________________

m(∠BCD) = ______________________________

m(∠DCE) = ______________________________

6. What is the difference between complementary and supplementary angles? Draw an example to illustrate your response.

7. Draw an angle according to the measure given in each box. Name the angle by its vertex and label what type of an angle it is. The first one is done for you.

a. m(∠ABC) = 180°  **∠ABC is a straight angle**	**b. m(∠QRS) = 35°**
c. m(∠MNO) = 90°	**d . m(∠EAT) = 100°**
e. m(∠XYZ) = 10°	**f. m(∠FUN) = 62°**

SUMMARY (What I learned in this section)

GEOMETRY 9

Name:_____________________________ Date:_______ Period:______

SECTION 9.2 Triangles

VOCABULARY

DEFINITION	EXAMPLE
Polygon:	
Vertex/Vertices:	
Equilateral Triangle:	
Isosceles Triangle:	
Scalene Triangle:	
Conjecture:	
Tessellation:	
Congruent:	
Triangle Sum Theorem:	
Right Triangle:	

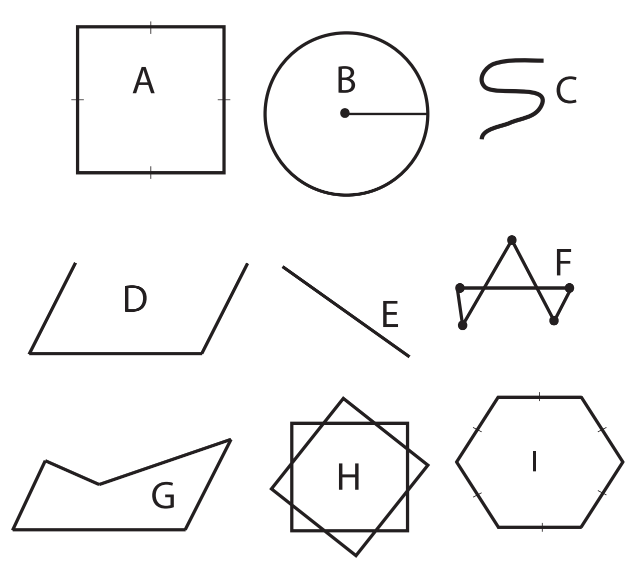
Hypotenuse:	
Legs:	
Pythagorean Theorem:	
Acute Triangle:	
Obtuse Triangle:	

Big Idea: How do you classify triangles?

EXPLORATION 1: POLYGONS

Consider the following figures and describe them by features that you observe. Identify common features as well as differing features among the shapes.

Figure	Characteristics	Polygon: P, Non-Polygon: NP # of sides
A		
B		
C		
D		
E		
F		
G		
H		
I		



Triangles are closed two-dimensional figures whose three sides are line segments. A triangle is one type of a ______________________________ .

Classify the shapes in Exploration 1 as a polygon or not a polygon. We often note the number of sides a polygon has. Include the number of sides each of the polygons has in Exploration 1.

Triangles can be classified by different properties - their size, shapes, and angles. In Exploration 2, you may discover different properties of triangles by making measurements of the triangles by lengths and angles.

EXPLORATION 2

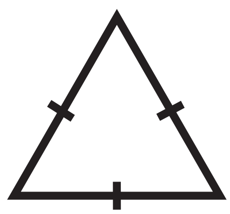
1. Draw a line segment that is five units long. Now draw two other line segments to complete a triangle. Repeat this process several times. What do you notice about the sum of the lengths of the other two sides relative to the length of the original side?

   ____

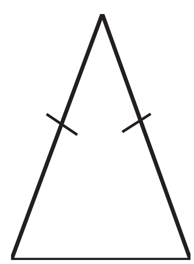
2. Make a triangle with two of the sides of equal length. Reflect on what you notice about each of these triangles. Measure and then make a rule about the angles of a triangle having two sides of equal length.

3. Make a triangle with all three sides of equal length. Reflect on what you notice about each of these triangles. Measure and then make a rule about the angles of a triangle with all three sides of equal length.

A way to indicate that lengths are equal in a given measure is using tick marks as indicated in the triangle below.



Classify the triangle: ______________________________

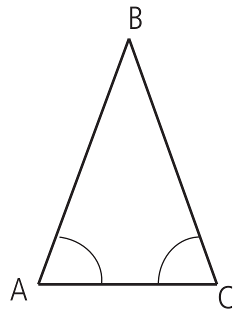


Classify the triangle: ______________________________

In the triangle above, did you notice that the angles opposite the equal sides are also equal? This is actually a property of all isosceles triangles:

- The angles opposite the equal sides are always equal

Just as we use a tick mark for equal lengths, there is a mark that is used to indicate angles of equal measure.



When two lengths or two angles have equal measure, then we say that they are **congruent** and have a special notation, ≅, to indicate congruence. For example, $\angle A \cong \angle C$.

Conversely, if two of the angles in a triangle are equal, then the sides opposite these equal angles will be equal and the triangle will be isosceles. These are properties that you will learn when you study geometry. Do you see why they might be true? ____________________

It is also possible that all three sides of a triangle might have different lengths. This type of triangle is called a ______________ ______________.

EXPLORATION 3: ANGLES IN A TRIANGLE

Draw a large triangle on a sheet of paper, using a straight edge. Color or label the three angles of the triangle with different colors. Carefully cut out the triangle. Next, cut the triangle into 3 triangular pieces, each including one angle from the triangle. Use the space below to glue your pieces together with the vertices touching. What is the sum of the three angles of the triangle? Compare your result with others.

The sum of the measures of the three angles in the triangle appears to be ________________ .

This is a _____________________________________ , because it is a statement we think might be true based on our observations, but we have not yet proved it is always true. Is there a way to give a convincing argument or proof of our conjecture?

____

You can use the angle measures to classify triangles as acute, obtuse, or right triangles. Use your knowledge of angles to fill in the table below:

Type of Triangle:	Characteristics:	Sketch:
Acute Triangle		
Obtuse Triangle		
Right Triangle		

EXPLORATION 4: TESSELLATIONS

On a separate sheet of paper, make a small triangle and cut it out. Trace your triangle below and name the angles *A, B, and C*. Use your cut-out to *tessellate* the paper or plane. A tessellation, or tiling of the plane with some shape, is a way of covering the plane with no gaps.

This tessellation can be used to show that the sum of the measures of the angles of any triangle adds up to ______________.

Each triangle in your tessellation above is ______________________ to your original triangle *ABC*, which means ____

______________.

PROBLEMS

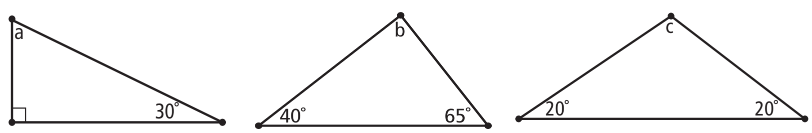
1. Draw two isosceles triangles with two equal sides of length 2 inches. Measure each of the angles. What do you notice about their measures? ____________________

2. Draw two isosceles triangles with two equal sides of length 2 inches. Measure each of the angles. What do you notice about their measures? ____________________

3. Draw two scalene triangles. Measure each of the angles. What do you notice about their measures? ____________________

4. A scalene triangle has three sides of different lengths. What can you say about the angles of a scalene triangle?
____

5. Find the measure of the missing angles:



m∠a= __________ m∠b =____________ m∠c=_______________

6. In the table below, use the side lengths to classify the triangle as equilateral, isosceles, or scalene.

Side A:	Side B:	Side C:	Classification:
4 cm	4 cm	2 cm	
5 in	5 in	5 in	
6 m	3 m	5m	
14 mm	14 mm	14 mm	
12.7 cm	12.7 cm	8.2 cm	
33 in	37 in	41.5 in	
0.6 m	0.6 m	0.6 m	
3 in	3 in	2 in	
4.5 cm	5.3 cm	5.5 cm	

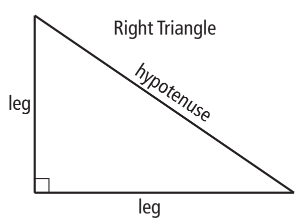
7. In the table below, use the angle measures given to find the missing angle measure. Then classify the triangle according to its angle measures as acute, obtuse, or right.

Angle A:	Angle B:	Angle C:	Classification:
90°	45°		
	100°	50°	
85°		55°	
130°	25°		
	90°	35°	
20°	95°		
	75°	55°	
60°	60°		
	89°	46°	

EXPLORATION 5: THE PYTHAGOREAN THEOREM

Triangles can be categorized by their angles. One kind of triangle is right triangle. Recall, a right triangle is a triangle with

____ .



The longest side of a right triangle is called the ______________________________ .
The right angle is __________________________ the
__________________________ . The two shorter sides are called the
______________ of the right triangle.

You will eventually learn a special theorem that relates the lengths of the legs of a right triangle to the length of the hypotenuse. This theorem, called the **Pythagorean Theorem**, enables you to find the length of any side of a right triangle if you are given the lengths of the other two sides. Looking at the triangle on the previous page, what ideas do you have about finding a missing length, if given the measures of two sides?

SUMMARY (What I learned in this section)

GEOMETRY 9

Name:________________________________ Date:_______ Period:______

SECTION 9.3 Quadrilaterals and Other Polygons

VOCABULARY

DEFINITION	EXAMPLE
Quadrilateral:	
Parallel:	
Parallelogram:	
Rectangle:	
Square:	
Trapezoid:	
Rhombus:	
Opposite Angles:	
Regular Polygon:	

Big Idea: How do you classify polygons? What are the attributes of quadrilaterals?

EXPLORATION 1: PREFIXES AND POLYGONS

"Tri" in triangle gives us a clue about the number of sides in that polygon. Similarly, "quad" in quadrilateral refers to its four sides. Other prefixes provide clues to the number of sides a polygon has. Complete the table below using the prefixes to help you name the polygons.

Prefix:	# of sides:	Name of Polygon:	Sketch:
Tri			
Quad			
Penta			
Hexa			
Octa			
Deca			
Dodeca			
	n		

All **quadrilaterals** have the common characteristic that they are polygons with four sides. Just as with triangles, quadrilaterals can be classified by properties of sides and angles. We first observe an important property of lines and line segments.

Let's think carefully about what seems like a simple concept, the idea of "parallel" lines. The question is how you decide whether two lines are actually parallel. In fact, what does it mean to say that they are parallel in the first place?

____

____

In the space below, we will construct a diagram of the relationship among various types of quadrilaterals. Follow the instructions listed below your workspace.

At the top of your workspace, write the word "Quadrilateral". Draw 2 arrows pointing downward and away from the word "Quadrilateral", being sure to give ample space between the arrows. Below one arrow, draw and label a trapezoid. From the other arrow, draw and label a parallelogram. From the parallelogram, draw two arrows branching off the base and connecting with a rectangle at the end of one arrow and a rhombus at the end of the other.
Lastly, from the base of the rectangle and rhombus, draw arrows pointing to a square. Make sure all of your figures are labeled.

The diagram you just drew is called a flowchart. Look at your flowchart and summarize your observations in three sentences:

____

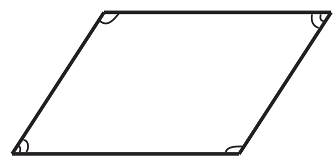
____

____

EXPLORATION 2: OPPOSITE ANGLES

Draw three different parallelograms in the space below:

Measure the angles in each parallelogram making a notation Do you remember how we used tick marks to show congruent sides in the first section of this chapter? We can also show that *angle measures* are congruent in a figure by drawing an arc inside the congruent angles. See the example below:



Go back to the parallelograms that you drew above and make the appropriate congruency marks. What do you notice about the marks you made in your parallelograms? ______________

____

____

Pay special attention to the angles opposite of each other. What do you notice? _______________

____

____

Now, look at the consecutive angles in each parallelogram. You should see a special relationship that we discussed in section 1. Write your observation here: ______________________

____

____

In each parallelogram, add the measures of the angles and write the sum in the center of your figure.

Think for a moment of specific parallelograms: rectangles and squares. What is the sum of the measure of all angles in a square? __________________ In a rectangle? __________________

How do those measures compare to the sum of angles you found in your own examples above?

Let's see how parallelograms compare to trapezoids.

Draw a trapezoid and a parallelogram on the grid paper below. Label the four angles in each of your quadrilaterals. Use your protractor to carefully measure the four angles in each shape and record the measures.

What do you notice in your findings?

____

Make a conjecture about the sum of angles in quadrilaterals: ______________________________

____

____

EXPLORATION 3: REGULAR POLYGONS

Do you remember the polygon exploration from section 2? In that exploration, you classified figures as polygons or not polygons. We know that the definition for polygon is ____

______________________.

A polygon is a **regular polygon** if

____ .

For example, an isosceles triangle is a polygon, but *not* necessarily a regular polygon. An equilateral triangle is an example of a regular polygon because

______________________________________ .

Write your own example of a polygon and a regular polygon:

The following activity will give you a chance to check your understanding of polygon classification.

Polygon Practice

Practice finding the angle measures of the following polygons.

1. What is the sum of the angle measures of a triangle?	2. What is the sum of the angle measures of a quadrilateral?	3. A quadrilateral has angles that measure 90˚, 100˚ and 120˚. What is the measurement of the fourth angle?
4. A parallelogram has opposite angles that measure 100˚. What is the measurement of the other angles?	5. A triangle has angles that measure 40˚ and 90˚. What is the measurement of the third angle?	6. An isosceles trapezoid has an angle that measures 45˚. What are the measurements of the other three angles?
7. What are the angle measures of a rhombus if one of its angles is 25˚?	8. The sum of the angle measures of an equilateral triangle is 180˚. What is the measure of each angle?	9. The sum of the angle measures of a regular hexagon is 720˚. What is the measure of each angle?
10a. Suppose four angle measures are 25˚, 75˚, 50˚ and 30˚. Can the measures of these three angles be used to form a quadrilateral? ________________ Justify your answer.		

EXPLORATION 4: POLYGON RIDDLES

Use the GEO-pieces found at the end of the riddles to answer each one.

I am a quadrilateral with one pair of parallel sides. ______________________

I am a triangle with no equal sides. ______________________

I am a quadrilateral with 2 pairs of parallel sides. ______________________

I am a three-sided polygon with one right angle. ______________________

I am a parallelogram with four equal sides and my opposite angles are congruent. ______________________

I am a polygon with four sides, four right angles and the sum of my four angles is 360 degrees. ______________________

I am a polygon with three sides and the sum of my angles is 180 degrees. ______________________

I am a polygon with exactly one obtuse angle. ______________________

I am a 5-sided polygon. ______________________

I am an 8-sided polygon. ______________________

I am a three-sided polygon with two congruent sides. ______________________

I am a triangle with equal sides and equal angles. ______________________

I am a 3-sided polygon with 3 acute angles. ______________________

I am a rectangle but a rectangle is not always the same as me. ______________________

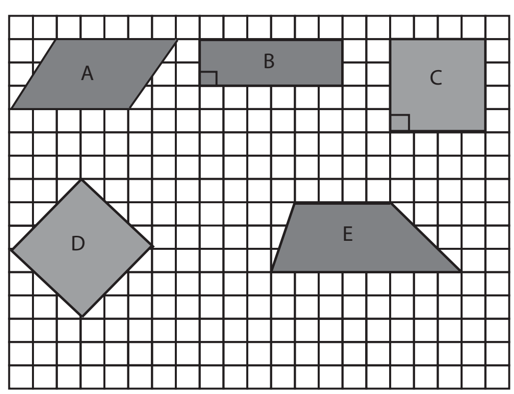
I am a parallelogram with only equal sides. ______________________

Now, go back to the GEO-pieces and label the Regular Polygons "RP".

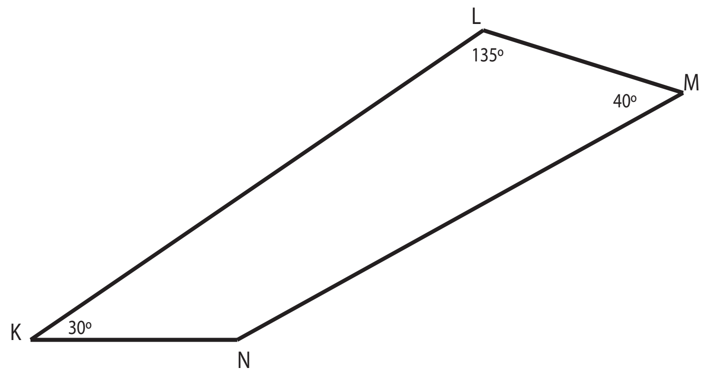
PROBLEMS

1. Classify the following quadrilaterals with the appropriate term or terms that apply.

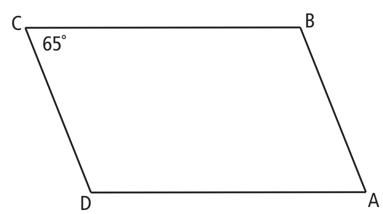
Figure:	Classification:
A	
B	
C	
D	
E	



2. Without using a protractor, find the measure of $\angle N$. Show how you arrived at your answer.



3. Find the measure of the missing angles in the parallelogram ABCD. Show how you arrived at your answers.



4. Determine the sum of interior angles of a regular pentagon. Explain how you arrived at this measure.

5. In the space below, show how a quadrilateral can be divided into two triangles. Using the graphic that you create, write your thoughts on how the sum of angles in a triangle compares to the sum of angles in a quadrilateral.

____

____

6. In the table below, use the angle measures given to find the missing angle measure in each quadrilateral.

Quadrilateral	Angle A:	Angle B:	Angle C:	Angle D:
M	60°	75°		80°
N	68°	118°	80°	
O	68°	126°	106°	
P	115°	65°	65°	
Q		110°	65°	110°
R	50°	135°		40°
S		35°	70°	145°
T	60°	100°	70°	

SUMMARY (What I learned in this section)

GEOMETRY 9

Name:________________________________ **Date:**_______ **Period:**______

SECTION 9.4 Perimeter and Area

VOCABULARY

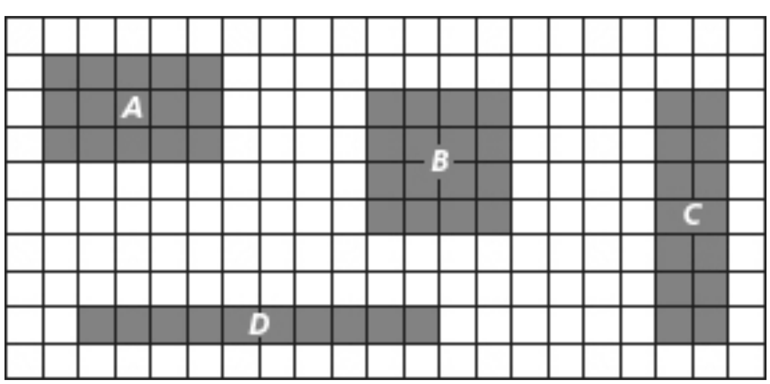
DEFINITION	EXAMPLE
Perimeter:	
Area:	
Height:	
Base:	
Trapezoid:	

Big Idea: How do you find perimeter and area of triangles and quadrilaterals?

How do you find the area of a 3 cm x 4 cm rectangle? How is this different from finding the distance around a 3 cm x 4 cm rectangle?

EXPLORATION 1: AREA AND PERIMETER

Among rectangles *A, B, C* and *D*, which is the biggest? Explain your answer.



There are several ways to think about what "biggest" means. One way to measure "biggest" is to find the area by counting the number of unit squares that are needed to cover each figure.

1. What are the areas of rectangles *A*, *B*, *C* and *D*?
2. Which one has the largest area?
3. Does this agree with the rectangle you chose?

Another way to measure the size of a rectangle is to add the lengths of all the sides. This sum is called the **perimeter**. Its name comes from the Greek words *peri*, meaning "around," and *metron*, meaning "measure."

4. What are the perimeters of the 4 rectangles?
5. Which one has the largest perimeter?
6. Does this agree with the rectangle you chose?

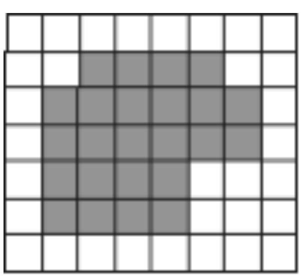
The perimeter of a rectangle is the sum of the lengths of the four sides. You can write this as perimeter of the rectangle $P = L + L + W + W = 2L + 2W = 2(L + W)$.

Notice that squares are special rectangles whose sides are all the same length. If we call the side length as *s*, then the perimeter of the square, $P = s + s + s + s = 4s$.

Just as you found the perimeter of a rectangle by adding the lengths of the four sides, you can find the perimeter of any polygon.

If the units of length and width are inches, then the perimeter is measured in inches. Area, however, is measured in square units. It can be labeled as square units or units^2.

EXAMPLE 1

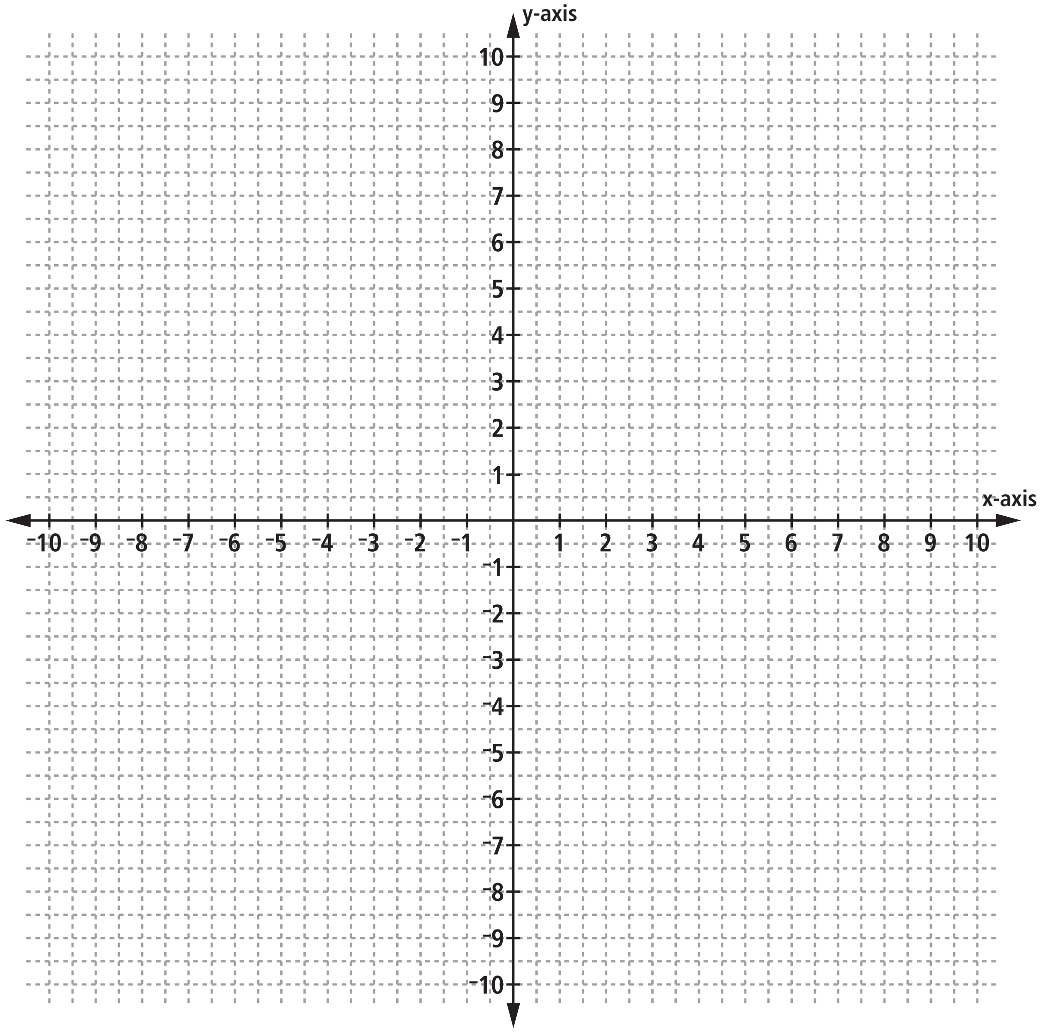


What is the area of the shaded region above? (Remember to record your answer in square units.)
____

What is the perimeter of the shaded region? _____________

EXPLORATION 2: AREAS OF RECTANGLES

Draw a 2 x 6 rectangle on the grid below.

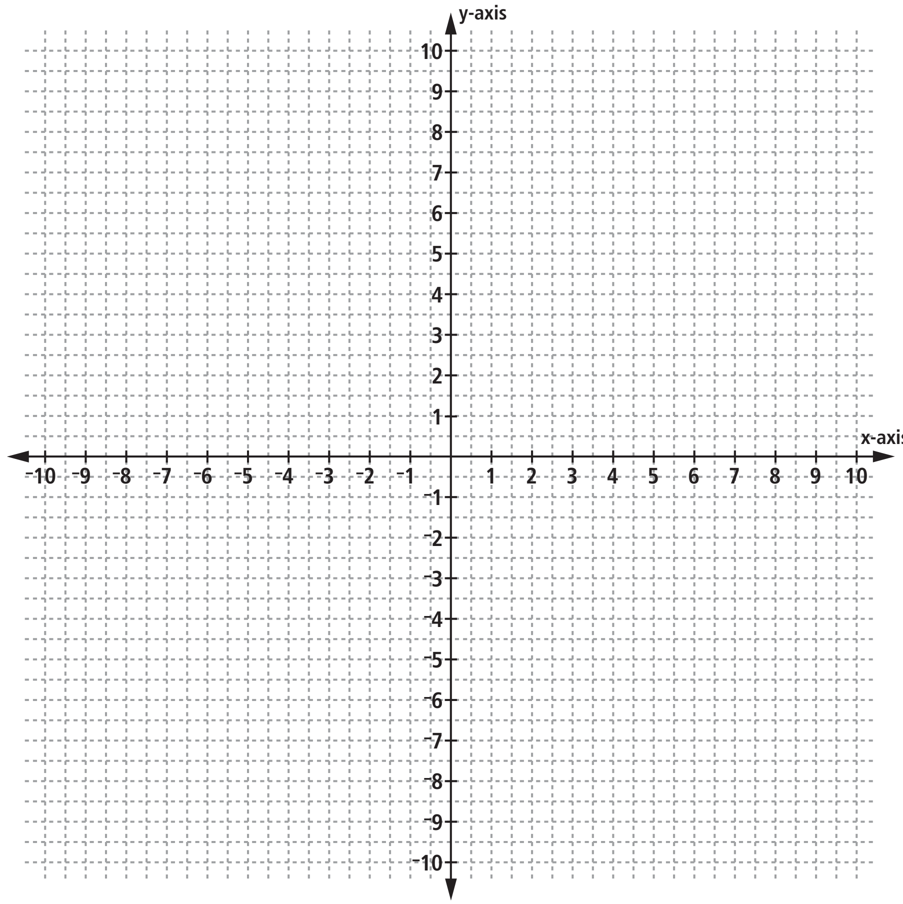


What is its length? _______ What is its width? _______

Now remember that 2 and 6 are dimensions that are multiplied together to give the area of this rectangle. The formula that describes area is Area = length · width or A = L · W.

Can you use this formula to find the area of a square?

Draw a 6 x 6 square.



If you use the formula $A = L \cdot W$, you would see that $6 \cdot 6 = 36$ square units. Because the square is a special type of rectangle, with four equal sides, it has its own formula.

Make a conjecture about how the formula for area of square is different from the area of a rectangle?

____

____

The formula for the area of a square is ____________________.

EXAMPLE 2

Sketch a square with a side length of 9 cm.

a. What formula will you use to find the area of this square? __________________________

b. What is its area? __________

EXPLORATION 3: AREA OF A PARALLELOGRAM

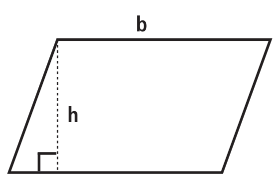
Let's consider another type of quadrilateral – a parallelogram. Using your own words, define a parallelogram:

1. Using a piece of grid paper, draw a parallelogram. Make sure the longer side of the figure is on one of the grid lines.

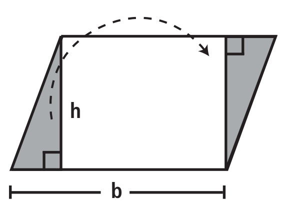
   a. Measure and record below the length of each of the sides and the measure of each angle.

   b. What do you observe?

2. Label one of the horizontal parallel sides of the parallelogram the **base**, with length *b*. To find the height, draw a line segment between the two bases perpendicular to each base. The **height**, *h,* is the length of the perpendicular distance between the two bases. Notice that the height in parallelograms, unless they are rectangles, is not the same as the length of either of the two non-horizontal sides



3. To find the area of the parallelogram, we will begin by cutting the parallelogram apart and reassembling the parts to form a rectangle, as illustrated below.

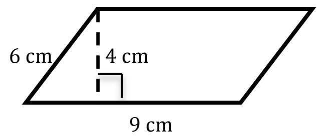


4. When reassembled, the parallelogram creates a rectangle with a length, or base, *b*, and width, or height, *h*. Using or knowledge of the area of a rectangle, we can see that the formula for the area of a parallelogram is:

$$A = b \cdot h \quad \text{or} \quad A = bh$$

EXAMPLE 3

Find the area and perimeter of the parallelogram below.

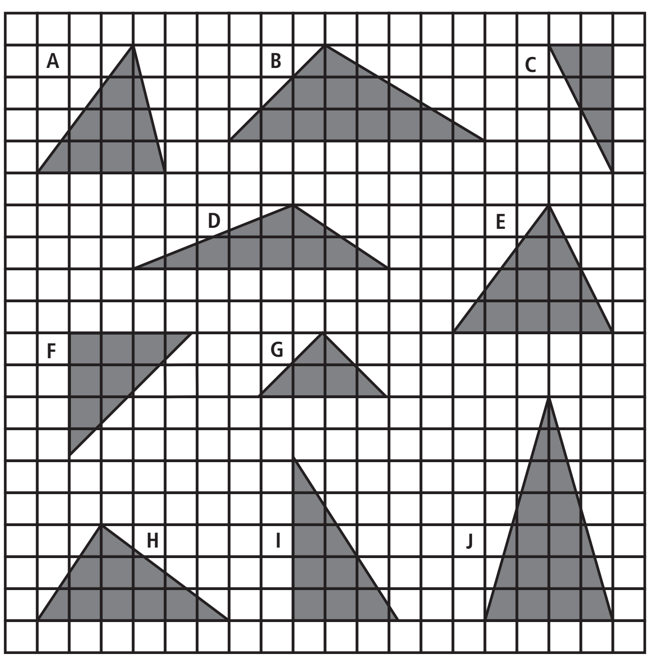


a. What is the area of this parallelogram? Show your work, beginning with writing out the formula for finding the area of a parallelogram.

b. What is the perimeter of this parallelogram? Show your work, beginning with writing out the formula for finding the perimeter of a parallelogram.

EXPLORATION 4: AREA OF TRIANGLES

Using the grid below, estimate the area of each triangle. The bottom is usually called the **base** of the triangle. Sometimes you may have to rotate the triangle to find its base.



A = __________

B = __________

C = __________

D = __________

E = __________

F = __________

G = __________

H = __________

I = __________

J = __________

a. How did you compute the areas of each triangle?

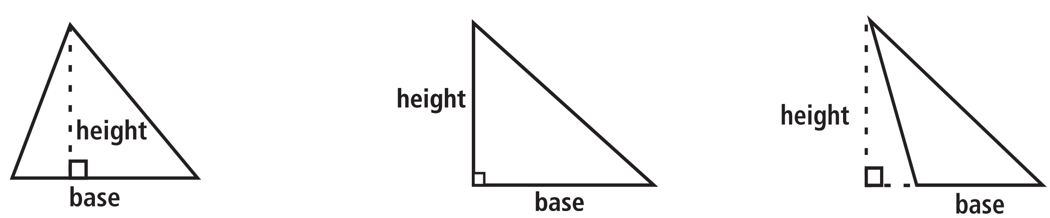
b. What patterns did you notice? Explain.

c. When you found the sum of the angles in a triangle in Section 9.2, you pasted two triangles together to form a four-sided figure. Using the triangles above, make a copy of each triangle and paste it together with the original triangle. What shape do you get? Use this process to find a rule for the area of these triangles.

What shape is formed when you take any triangle, copy it exactly, and put the two triangles together?

____

So use the formula for the area of the parallelogram and take one-half of it to compute the area of the triangle. Be careful in identifying the base and the height of the triangle. The base must be a side of the triangle, and the height, or **altitude**, must be perpendicular to the base, or an extension of the base, and be drawn from the vertex opposite the base.

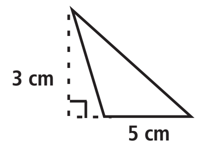


The formula for the area of a triangle with height h and base b is: ____________________________.

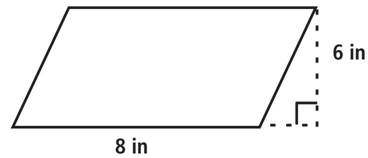
EXAMPLE 4

Find the area of the following polygons.

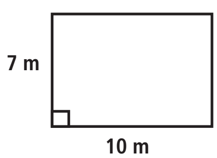
a.



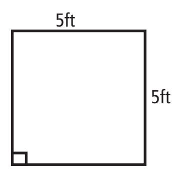
b.



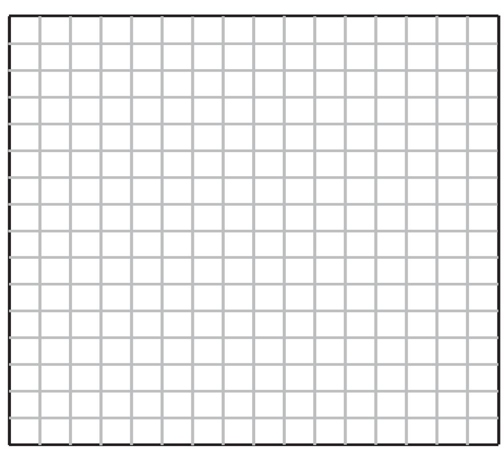
c.



d.

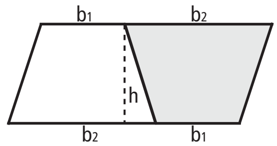


1. Draw a trapezoid towards the left with base no larger than 8, on the grid below.



2. Now draw an upside down congruent trapezoid next to the first. What is the resulting shape? ______________________________

3. The two trapezoids should form a large parallelogram similar to the picture below.



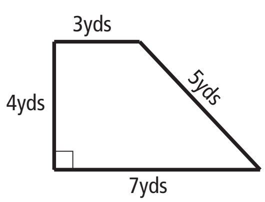
Using what you know about parallelograms, what is the area of the parallelogram? Because the parallelogram consists of the same trapezoid twice, what should the area of one trapezoid equal?

____

4. The formula for the area of a trapezoid is: ________________________________.

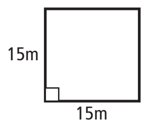
EXAMPLE 5

Find the area and perimeter of the following trapezoid:



PROBLEMS

1. Find the area and perimeter of each figure.

a.


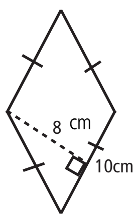
Area:

Perimeter:

b.

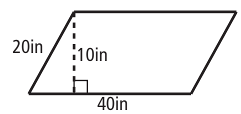

Area:

Perimeter:

c.


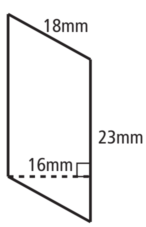
Area:

Perimeter:

d.


Area:

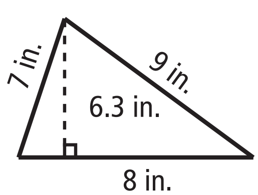
Perimeter:

e.


Area:

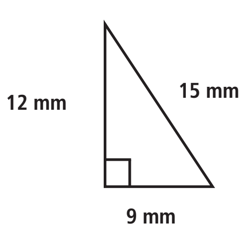
Perimeter:

2. Find the area and perimeter for each triangle.

a.


Area:

Perimeter:

b.


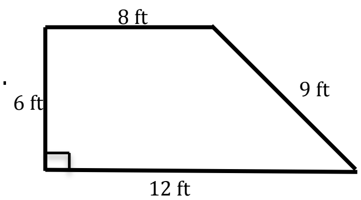
Area:

Perimeter:

3. Find the area and perimeter of the following trapezoids:

a.

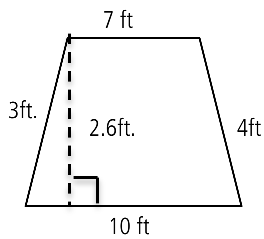
a.



Area = ________________

Perimeter = ___________

b.



Area = ________________

Perimeter = ___________

4. An equilateral triangle has base lengths of 4 yards and 3 yards. What is its area?

5. A regular pentagon has side lengths of 3 cm. What is the perimeter of the pentagon?

6. Mia is carpeting her rectangular bedroom. The dimensions of the room are 12 ft. x 15 ft. Determine how many square yards of carpet she will need to purchase to cover the entire floor.

SUMMARY (What I learned in this section)

GEOMETRY 9

Name:__________________________________ Date:________ Period:______

SECTION 9.5 Circles

VOCABULARY

DEFINITION	EXAMPLE
Center:	
Diameter:	
Circumference:	
π:	
Constant:	
Coefficient:	
Radius:	
Chord:	
Semicircle:	

Big Idea: What are the important attributes of circles? How do we compute area and circumference of circles?

Everyone has seen circles of various sizes, but what is the definition of a circle? How do you draw a circle? Try to describe a circle to someone without using the word "circle."

EXPLORATION 1: DRAWING A CIRCLE

How do you draw a circle? Once you have drawn a circle, write directions someone could follow to draw a circle.

Now, state your definition of a circle.

In general, one way to draw a circle is to first draw and label a point. Do so in the space below: (Hint: you need to make your point in the middle of the space!!!)

Take a length of string, *r* units long, placing one end on the point and attach the other end to your pencil. Stretch the string to its full length and draw the circle with your pencil. A circle is named by its center point. What is the name of your circle? ________________________ What can you say about the distance of any given point on your circle to the center point?

The fixed distance, *r*, from the center point to any point on the circle is called the _____________. All line segments that can be drawn from one point on the circle to another is called a ____________________. A line segment connecting two points on the circle AND passing through the center point is a special chord called a __________________________ . The length of the diameter is equal to the length of 2 ____________________. Radii is the plural form of radius.

In the circle you drew above, draw and label a radius, chord, and diameter.

Notice that the _________________________ cuts the circle in half, forming two semi-circles. Also, notice that it is the longest line segment that can be drawn from one ____________________ to another on the circle.

Are all diameters considered chords? ________________________

Are all chords considered diameters? ________________________

The distance around the circle is called the ______________________________, and is like the perimeter of a polygon.

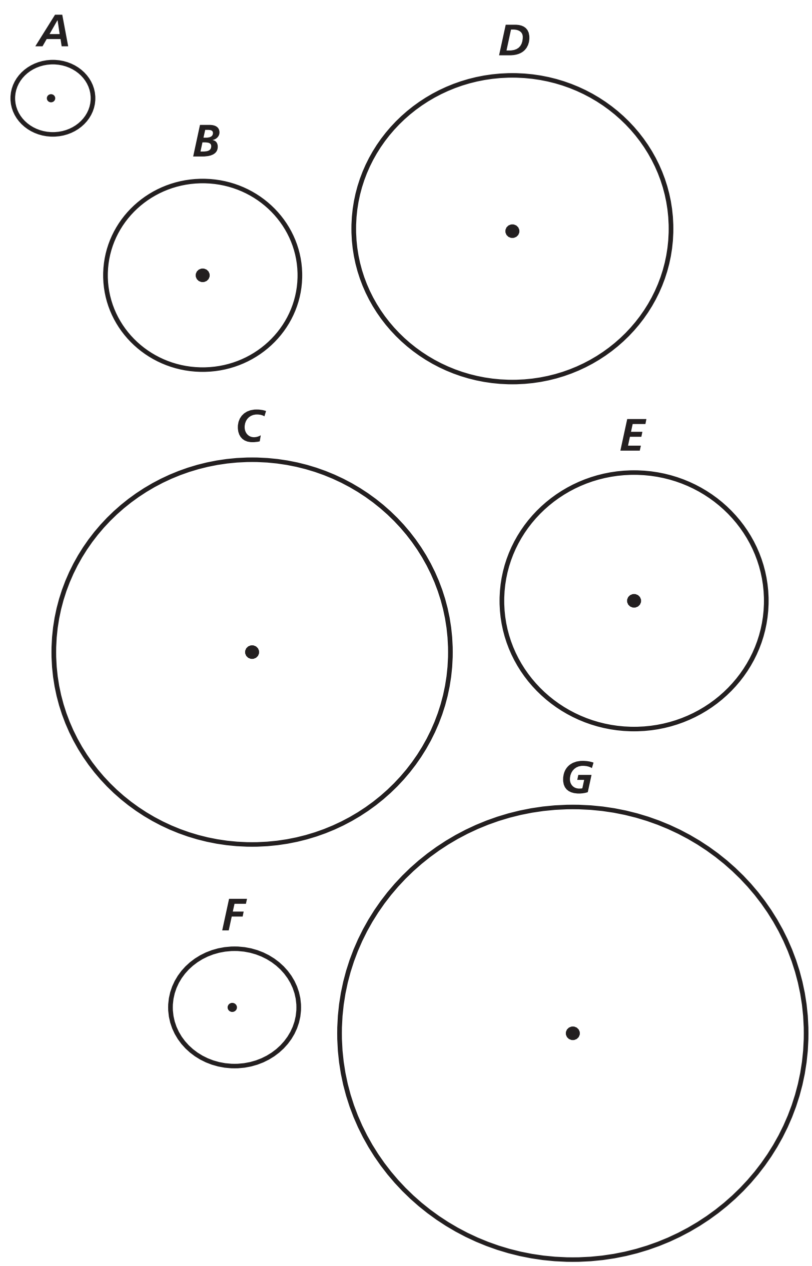
Is a circle a polygon? __________ Why or why not?

____

Label the circumference of your own circle.

EXPLORATION 2: RELATIONSHIPS

Use the circles below to complete the activity. Using a ruler and piece of string, carefully measure the radius and circumference of each circle. Record your results in the table that follows.



Circle:	Radius *(r)*:	Diameter *(d)*:	Circumference *(C)*:	$\frac{C}{d}$:
A				
B				
C				
D				
E				
F				
G				

Do you notice a relationship between the radius and the diameter? ________________________________

Using the variable, *d,* to represent the length of the diameter, express the diameter in terms of the radius, *r*.
____
____

What is the relationship between the circumference of a circle and its diameter? ______________
____
____

Compute the ratio of the circumference to its diameter. What do you notice about the ratios?
____

____
____

The ratio you computed approximates the exact ratio of the circle's circumference to its diameter, the number pi, written as the Greek letter ________ . This ratio, $\frac{C}{d} = \pi$, of the ______________________________ to ________________________________ is the same regardless of the size of the circle.

What would happen to the ratio $\frac{C}{d}$, if one of the circles was scaled by a factor of 2, making the radius twice as large?
____
____

When the radius doubles, what happens to the circumference?

____

Let's summarize what we have learned. Use the variables C for circumference, d for diameter, and r for radius and write an equation finding each in terms of the others.

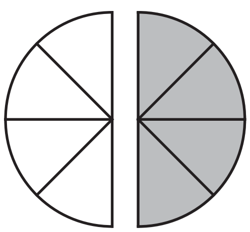
$C =$ ______________________________ $d =$ ______________________________

$r =$ ______________________________

EXPLORATION 3: AREAS OF CIRCLES

What is the area, A, of a circle whose radius is 1?

To do this exploration, you will need to draw your circle on a separate sheet of scratch paper. Draw a circle with a radius of 1 unit and circumference of 2π units. Cut the circle in half, and then continue to cut each half into as many small pie slices of equal size, as illustrated below:



Take the slices from one half of the circle and lay the points of the slices along the line drawn for you:

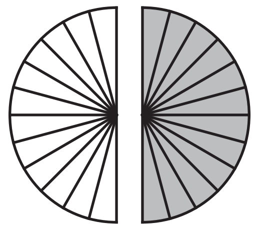
Do the same with the other half of the circle, filling in the spaces. (All of your slices should be on the same side of the line.) You may glue your slices to this page for your records.

What shape does this look like? ____

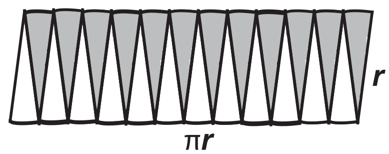
Imagine if you cut each slice in half again and repeated the process of laying the slices on the line. What shape do you think this would resemble? ____________________________________

If this cutting process continued infinitely, the area of the circle with radius of 1 unit would approximate the area of a rectangle with length π and width 1 unit.

What happens to the area of the circle when its radius is a number r? One way to visualize this is to create slices in the circle with radius r, like the previous process with radius 1.



Cut the circle into two equal semicircles as you did in the unit circle and fit one semicircle into the other semicircle.



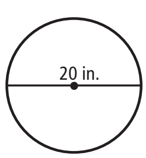
What is the length of this rectangular shape? What is its width? What is the area of the rectangle?

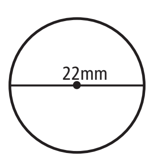
In this rectangle the length is πr, which is half the circumference $2\pi r$ and the width is r. The area of the rectangle is length times width or $\pi r \cdot r$ or πr^2. Any area is measured in square units. So if r is measured in inches, $r \cdot r$, or r^2, is measured in square inches. To summarize:

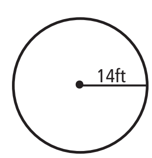
Formula 9.5: Area of a circle
The area of a circle with radius r is $A = \pi r^2$ square units.

PROBLEMS

1. Find the circumference of the following circles with the given radius or diameter. Find your answer in terms of π, then use 3.14 as an approximation for π.

a. 

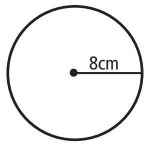
b. 

c. 

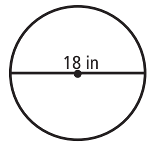
$C =$ ________________ $C =$ ________________ $C =$ ________________

$C \approx$ ________________ $C \approx$ ________________ $C \approx$ ________________

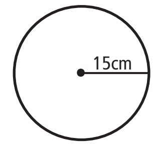
d.



e.



f.



$C =$ ________________ $C =$ ________________ $C =$ ________________

$C \approx$ ________________ $C \approx$ ________________ $C \approx$ ________________

2. Using the information provided in the table, complete the missing cells.

Radius, *r*	**Diameter, *d***	**Circumference, *C***
9 cm		
	16.4 ft.	
2.5 in		
	42.8 mm	
	65 m	

3. How does the radius relate to the diameter?

____

____

To the circumference?

____

____

4. Find the area of each of the circles and complete the table below:

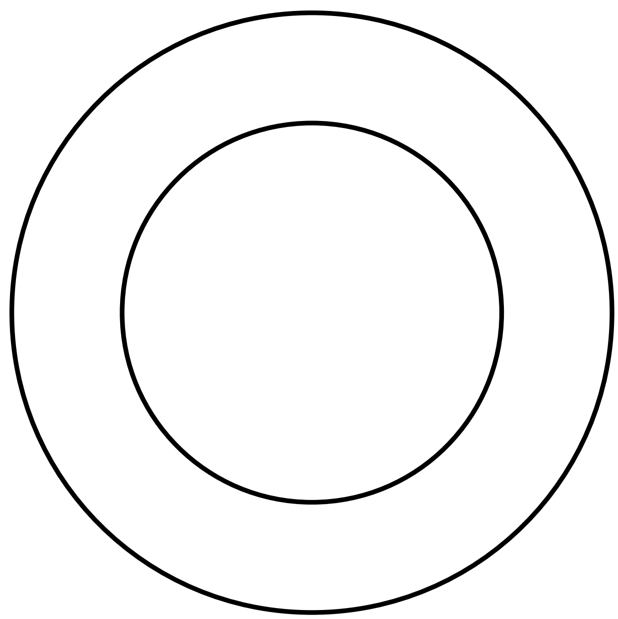
Radius, *r*	**Diameter, *d***	**Circumference, *C***	**Area, *A***
5 in			
	1.4 mm		
3 ft.			
	6.4 mm		
1 m			

5. Draw a circle and label the following: circumference, chord, radius, and diameter.

6. A spoke from a unicycle wheel is 10 ft. How many feet will the unicycle travel after 5 rotations?

   The unicycle will travel ____________________ feet after 5 rotations.

7. Ricardo draws a circle with a 6-inch diameter. He draws another circle inside the first, this one with a 5-inch diameter. He is going to paint between the two circles and wants to know the area he has to cover. Use the diagram below to assist you in solving this problem. Begin by shading the area Ricardo wants to paint.



Ricardo will paint ________________ square inches.

SUMMARY (What I learned in this section)

Name:________________________________ Date:_______ Period:______

SECTION 9.6 Three-Dimensional Shapes

VOCABULARY

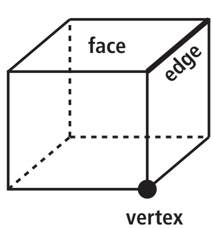
DEFINITION	EXAMPLE
Polyhedron/Polyhedra:	
Vertex/Vertices:	
Faces:	
Edges:	
Prism:	
Bases:	
Rectangular Prism:	
Lateral Faces:	
Pyramid:	
Apex:	

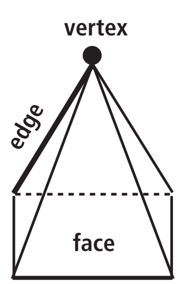
Cone:	
Cylinder:	
Sphere:	
Regular Shapes:	
Cube:	
Volume:	
Nets:	

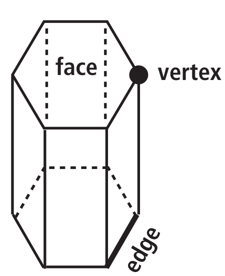
Big Idea: How do you classify three-dimensional shapes? How do you find volumes of prisms?

A basic kind of three-dimensional figure is called a ______________________________. This word comes from comes from the Greek words *poly*, meaning __________________, and *hedra*, meaning ________________________.

Each face of a polyhedron is a __________________________. The vertices of the polygon are the ____. The edges are the ________________________ of the ___________________ that are also the _______________________________________ that join the vertices. Look at some examples below.



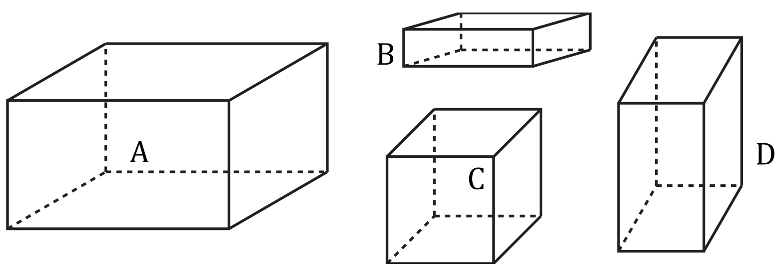




A box shape is the most common type of polyhedron called a ______________________________. In this shape, two of the faces, called ____________________________, are parallel and congruent. _________________ are named by their __________________________. In the case of a box, the polyhedron is a ____, because the bases are __________________________. The faces that connect the two bases are parallelograms, and in this case rectangles. They are called ____.

EXPLORATION 1: PRISMS

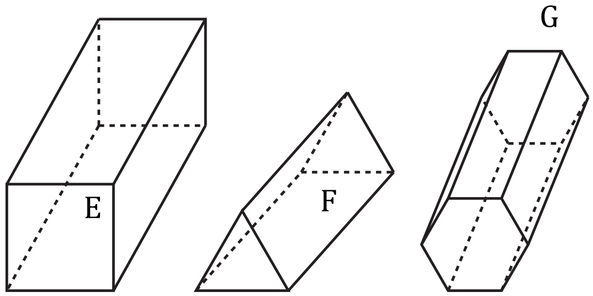
As noted above, prisms are named by their bases. **Rectangular prisms** have rectangles as their bases. Although the faces of prisms are not always rectangles, in rectangular prisms, all of the faces are rectangles. Look at the rectangular prisms shown below:



You will need five different colored pencils, pens, or highlighters.
Choose one color to draw the vertices on each prism. Count the vertices and record your information in the table.
Using a different color, draw a line over the edges. Count and record the number of each in the table.
You probably can guess what you should do next. Continue to choose a different color to shade and count the faces, bases, and lateral faces. Record your information in the table.

Figure	Faces	Edges	Bases	Lateral Faces	Vertices
A					
B					
C					
D					

Pictured below are three different prisms:



______________ ______________ ______________

______________ ______________ ______________

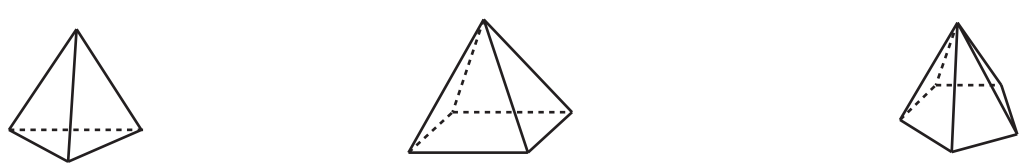
______________ ______________ ______________

On the lines below each shape state what shapes are the base faces, what shape are the lateral faces, and name each prism. Repeat the same process as before to fill out the table below.

Figure	Faces	Edges	Bases	Lateral Faces	Vertices
E					
F					
G					

EXPLORATION 2: PYRAMIDS

Pyramids are another type of ____________________________. Unlike a prism, pyramids only have one ________________________, which is a ________________________________, and triangular faces that meet at a point called the _________________. Like prisms, pyramids are named by their ______________________. Identify the names of each pyramid below:



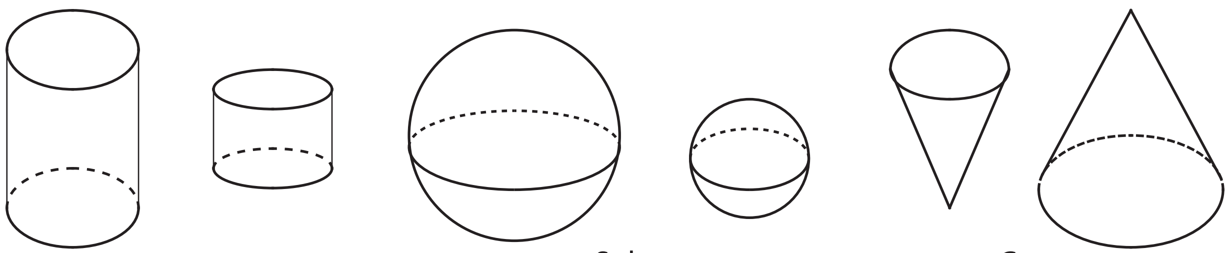
______________ ______________ ______________

Compare and contrast prisms and pyramids on the lines below:

____

____

EXPLORATION 3: CONES, CYLINDERS AND SPHERES



Other common three-dimensional shapes include cones, cylinders and spheres.

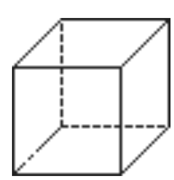
Notice that ____________________ are related to pyramids, but have a ____________________________ base. A __________________________ in a similar way is related to a prism, but has ________________________________ bases. A ________________________ is a three dimensional version of a circle, a figure formed by all points of a fixed _______________________ from a fixed point, called a ________________________________.

In the chart that follows, write some real-world examples of each figure.

3-D Figure	Examples
Cylinder	
Sphere	
Cone	

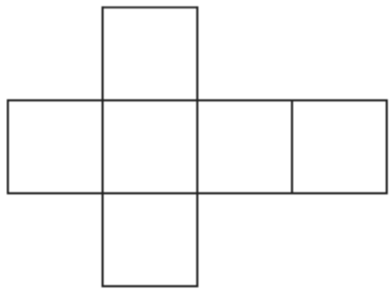
EXPLORATION 4: CUBES

A cube is a ______________________________ rectangular prism because each of its ______________________ has _________________________ sides and angles. All the cube's faces are _________________________________. The cube is the simplest three-dimensional shape to measure having two parallel congruent square _____________________________ connected by four perpendicular congruent square lateral _________________________.



A cube one unit long, one unit wide, and one unit high has a volume of one _____________________ ________________________. Label the length, width, and height of the cube.

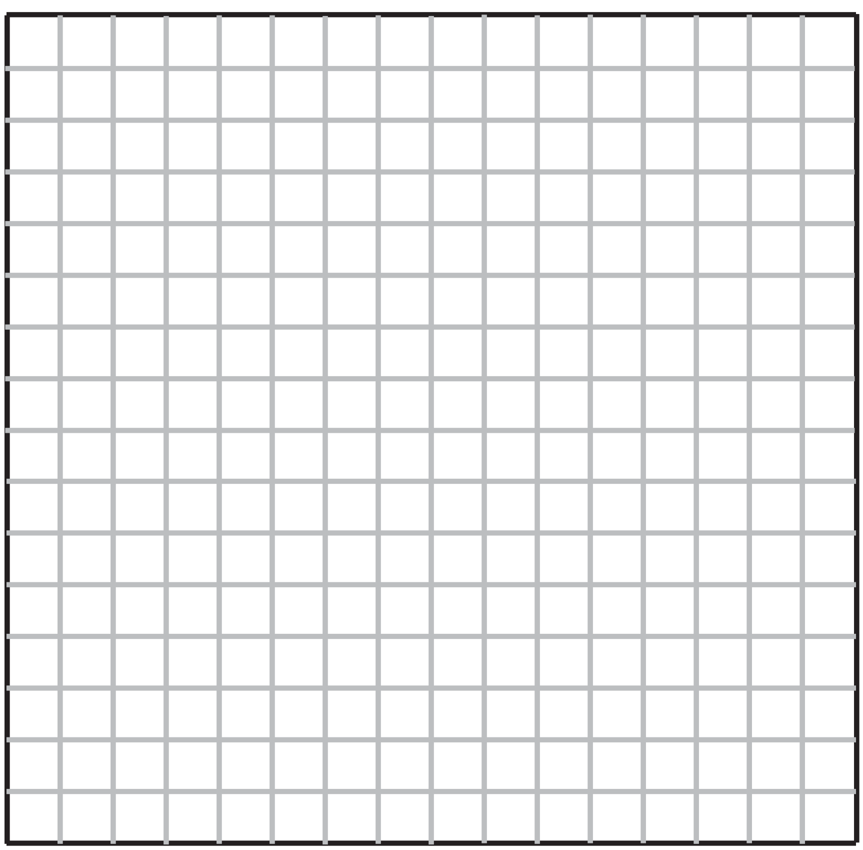
Remember that each face of a three-dimensional shape is a __________________________. In section 4, you learned about finding the area of a two-dimensional figure. If you were to cut along some of the edges and flatten the cube to create a net, it could look like the diagram pictured below.



Color two of the squares that would form the bases. Keep in mind the location of the squares when folded into a cube. Color the lateral faces. Figure the area of each square and write it inside each one. Add the area of each square together. This will give you the surface area of the cube. You will learn more about Surface Area in the part 2 book.

How many square units is the net? __________________________

Can you think of other ways in which the cube can be cut into a net? Sketch your ideas using the graph paper given:

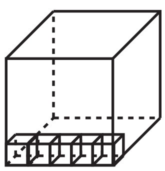


EXPLORATION 5: VOLUME

Using centimeter cubes, build a layer of length 2 cm and width 2 cm. Discuss the number of cubes used. What is the area of the figure base? ____________________________

Now add an identical layer on top of the base. How many total cubes are used? ________________ To find the answer, you can take the answer for the number of cubes used in the base and multiply it by the number of layers. We now have: ______ · ______ · ______ = _________ cubic centimeters. Common abbreviations for cubic centimeters are cu cm and cm^3. This is the volume of the cube you constructed.

The _____________________ of a three-dimensional shape is measured by the number of cubic units needed to fill it. Consider a cube that is 5 units long, 5 units wide, and 5 units tall. How many cubic units would be needed to fill this shape?



Look back at Exploration 4 where you labeled the length, width, and height of a cube that measured one unit for each dimension. Imagine placing those cubes inside the larger cube as pictured above. It would take ______ of the unit cubes to make a lengthwise row. How many rows would you need to cover the surface? _________ How many rows would you have to stack to fill the cube to the top with the smaller unit cubes? ________ The product of these dimensions would give us the volume of the cube: ______ · ______ · ______ = _________ cubic units. Since we see a factor that is repeating, we could write this expression in exponential form as _______ = _______ cubic units.

What is the difference between the unit used to denote area and the unit used to denote volume?

EXPLORATION 6: CONVERTING UNITS

How many cubic inches are there in one cubic foot?

Begin this problem by creating a sketch of a cube and labeling its dimensions. Did you label in feet or inches? _______________ Why?

____

The problem states that we have a cube that measures one cubic foot, but it is asking us how many cubic *inches* there are in that one cubic foot. Write the inch to foot conversion:

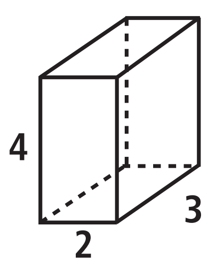
____

We have found that the volume of a cube is found by multiplying side times side times side, $s \cdot s \cdot s$, or s^3.

To start, write your formula using exponential notation, substituting *12 in.* for *s* in the formula: $s^3 = V$ in expanded form, substituting *12 in.* for *s* in the formula: $s \cdot s \cdot s = V$ Now, write the formula Multiply to find the product, which is the volume of the cube. Remember to write: $V =$ ______, using the appropriate units.	(Show your work here.)

EXPLORATION 7: VOLUME OF A RECTANGULAR PRISM

The rectangular prism below has edges that are 2, 3, and 4 units long.



How many unit cubes does it take to fill the box? The 2 x 3 base rectangles can be cut into _______ unit squares, and the height is ________. To find the volume, multiply the area of the base times the height: ______________________________________ Therefore, $V =$ ____________________.

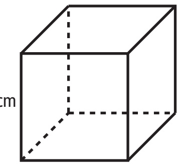
FORMULA 9.6: volume of a prism
The volume formula for a prism can be written by volume = area of base · height or $\boldsymbol{V = Bh}$ with B = area of the base and h = height of the prism. In particular, for a rectangular prism, $B = lw$ so $\boldsymbol{V = lwh}$.

PROBLEMS:

1. Fill in the table with the number of vertices, edges, faces, and a sketch of each shape.

Name of Shape:	Vertices:	Edges:	Faces:	Sketch:
Triangular Prism				
Rectangular Prism				
Pentagonal Prism				

2. Find the volume of the cube with sides of length of 6 cm.



3. What is the volume, in cubic feet, of a cube with side lengths of 6 inches? (Hint: Don't forget to convert!)

$V =$ ______________

4. A koi pond has dimensions 8 feet x 6 feet x 3 feet. Determine the volume of the pond.

The pond has a volume of ____.

5. If the volume of the form that makes a sugar cube holds 8 cm^3, what is the measure of the side length?

____

6. Every rectangular box made by the Bow-Wow Box Company has a length of 2 feet. Find the Volume of each box that the company makes.

Width, *(ft.)*:	**Height, *(ft.)*:**	**Volume:** (remember to use the appropriate unit)
1	**4**	
2	**6**	
3	**8**	
4	**10**	

7. Find the missing value, h, in the formula for the volume of a rectangular prism. Be sure to indicate the units for h.

   $12.5\text{ m} \cdot 5\text{ m} \cdot h = 187.5\text{ m}^3$

   $h =$ ________________

SUMMARY (What I learned in this section)

____

____

____

____

____

____

Name:____________________________ **Date:**________ **Period:**_______

CHAPTER 9: SPIRAL REVIEW

1. Complete the table with the appropriate equivalent metric measurement in each row:

Kilo-	Hecto-	Deca-	Base *m, L, g*	deci-	centi-	milli-
			15.2 *g*			
					14 *cm*	
						13.7 mL
				8 *dL*		
			62.5 *L*			
		23.9 *Dm*				
	39 *hL*					
165.7 *kg*						
5,532 *km*						
			1 *g*			

2. Fill in the missing information about various rectangles in the table below:

Length	Width	Area	Perimeter
5 in	3.2 in		
		24 m^2	20 m
7 cm		78.4 cm^2	
9.9 ft	12.5 ft		
	1.7 yds	3.74 yds^2	

3. Evaluate the following expression using the Order of Operations:

$$50 - 2 \cdot 16 \div 2^3 + 3$$

____

____

____

____

4. Suppose two angles are complementary, and the measure of one angle is 25.8°. What is the measure of the other angle?

Measure of other angle is ________________.

5. A right triangle has one angle measuring 35°. What is the measure of the missing angle?

The missing angle measures __________________.

6. Curtis has 16 racecars, 48 strips of racetrack, and 56 miniature road signs. He wants to separate the different items into as many identical groups as possible to give to his younger cousins. What is the largest number of identical groups Curtis can make using all these items

Curtis can make _______________ groups. Each group will have ______ racecars, _______ strips of racetrack, and _______ miniature road signs.

7. What is the sum of the angles in a triangle? _________ In a quadrilateral? __________

8. What is the volume of a rectangular box that is 22 inches long, 14 inches wide, and 10 inches tall? Indicate your units.

The volume of the box is __________________________________.

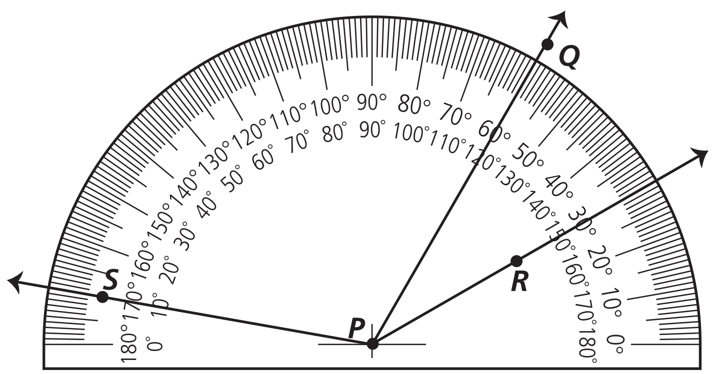
9. A complete revolution that a hula-hoop makes is about 108 inches. What is the approximate measure of the diameter?

The diameter is ________________________________.

10. Find the fraction, decimal, and percent conversion of each number. Show your work below the table.

FRACTION	DECIMAL	PERCENT
	0.76	
		89%
		105%
	0.246	
		23.5%

11. Find the measure of the angle indicated.



DATA ANALYSIS 10

Name:_________________________________ **Date:**_______ **Period:**______

SECTION 10.1 Measures of Central Tendency

VOCABULARY

DEFINITION	EXAMPLE
Data Analysis:	
Data:	
Data Point:	
Measure of Central Tendency:	
Range:	
Mean:	
Average:	
Median:	
Frequency:	
Mode:	

Dot Plot:	
Stem and Leaf Plot:	
Skewed:	
Outlier:	
Symmetric:	
Box Plot:	

Big Idea: How do you find measures of central tendency of a set of data? What are some ways data is represented?

Sets are useful for grouping interesting and related numbers. One such set is the height of all of the people in your class. In order to use these sets, we need to analyze the numbers, or data, in context. The first step in **data analysis**, the process of making sense of a set, is collecting data. In data analysis, the idea of a data set is slightly different from that of a set. Unlike regular sets, data sets can have repetition of elements, and the order or arrangement matters.

EXPLORATION 1: SUMMARIZING DATA

The table below indicates the names, height and ages of all the students in Ms. Rosenbush's 6th grade math class

Name	Height (in)	Age (months)
Sophia	52	113
Rhonda	51	112

Name	Height (in)	Age (months)
Edna	57	112
Danette	61	115

Name	Height (in)	Age (months)
Hesam	55	117
Eloi	62	110
Vanessa	58	113
Michelle	60	108
Mari	58	125
Calvin	56	129
Moises	57	124
Amanda	57	120
Hannah	55	131
Tricia	55	129
Kristen	57	130
Max	52	135

Name	Height (in)	Age (months)
Jim	50	142
Karen	57	136
Diane	49	138
Tiankai	58	138
Oscar	51	137
Jenny	60	138
Bence	59	142
Pat	53	134
Teri	59	135
Sally	57	139
Will	57	140

Try to find ways to summarize the information in the table so that you can share the results with a friend without showing her the whole table.

____

____

Would your strategy still work if there were 100 people in the survey? 1,000 people?

____

____

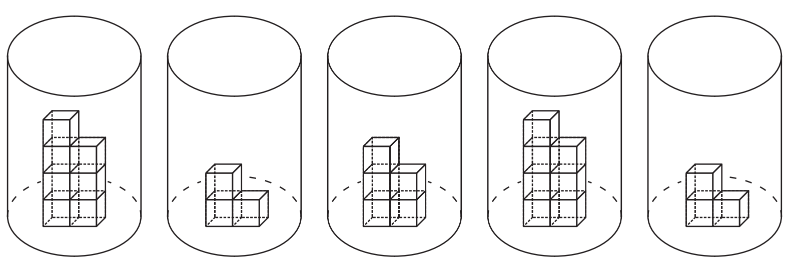
When analyzing numerical information, the entire collection of numbers studied is called the _________ and each individual piece of information is called a __________________.

A major goal of data analysis is to find a simple measure of data, called a ____________________, that summarizes or represents the majority of the data. Three common measures of central tendency are the ________________, __________________ and ________.

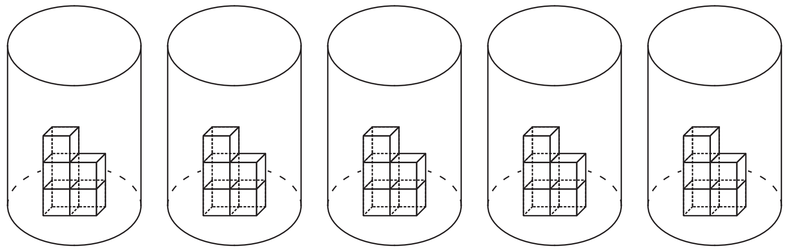
We are also interested in how spread out our data is. The ____________________, the difference between the largest and smallest values of the data, provides a simple measure of how much the data varies.

EXPLORATION 2: FINDING THE MEAN & MEDIAN

The **mean**, also called the arithmetic mean or average, is the sum of all the data values divided by the number of data points. For a visual example, suppose we have five containers, each containing a certain number of blocks:



These data can be grouped into a data set: {7, 3, 5, 7, 3}. We will notice the importance of the order of arrangement and the repetition. There are 25 blocks total. The mean number of blocks in a container is the number of blocks each container has if these 25 blocks are distributed evenly among the 5 containers: $\frac{25}{5} = 5$.

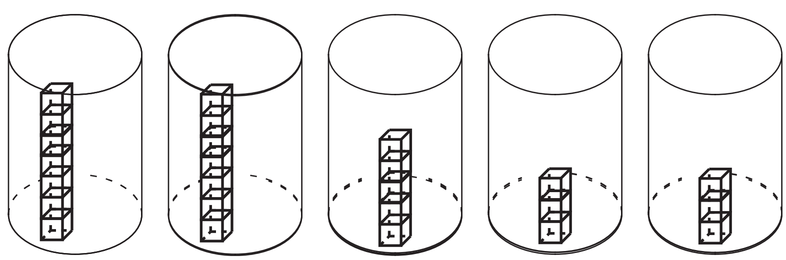


EXAMPLE 1

Find the mean of the following data set.

{91, 100, 83, 76, 37, 98}

The **median** is the value of the middle data point when the values are arranged in numerical order. If the data set has an even number of data points, the median is the average of the two middle values. To find the median value for the container example, order the data, with the largest number of blocks first and the smallest number last:



The median is the number of blocks in the middle, or third container with respect to the sorted ordering. The median is a helpful measure of central tendency because half of the values are less than or equal to the median and the other half of the values are greater than or equal to it.

EXAMPLE 2

Find the median of the following data set.

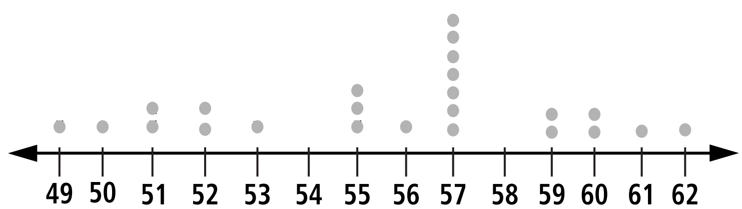
{91, 100, 83, 76, 37, 98}

Note that the mean depends on all the numbers in the data, but the median only depends on the value of the data point in the middle position. That does not, however, suggest that the mean is a better measure of central tendency than the median.

EXPLORATION 3: ILLUSTRATING DATA

Frequency is the number of times a data point appears in a data set. For example, if there are 4 people in class who are 56 inches tall, then the frequency of the height 56 inches in the class is 4. The **mode** is the value that occurs the most often in the data set. A set of data can have more than one mode. For the containers of blocks in Example 2, above, the modes are 3 and 7, because both appear twice. If no data points are repeated in a set of data, we say that the data set has no mode.

One way to illustrate the frequency of a relatively small data set is to use a **Dot plot**. The horizontal line represents the number line and a dot is placed above the numerical value for each time that value occurs in the data set, as in the example below.



EXAMPLE 3

Create a **Dot plot** to depict the heights of Ms. Rosenbush's students listed in the tables in Exploration 1.

Use your line plot to answer the following questions.

a. What is the mode of the data? ________________

b. What is the range of the data? _______________

EXAMPLE 4

Another way to view a data set is to use a **Stem and Leaf Plot**. The leaf is usually the last digit of the numbers in the data set and the stem is the rest of the numbers to its left, arranged in a vertical numerical order. Let's try one.

Create a stem and leaf plot for the heights of Ms. Rosenbush's students from the table in Exploration 1.

Stem	Leaf

As you can see, the stem and leaf plot is useful in showing the frequency of a number in the data set. It can also be useful when looking for the median.

PROBLEM

You are given a data set represented by the following stem and leaf plot:

Stem	Leaf
10	0
9	6442
8	98777553
7	98841
6	73

Use the information to determine the following, if possible, and round any value to the nearest one:

1. The mean of the data set.
2. The median of the data set.
3. The mode of the data set.
4. The range of the data set.
5. A dot plot of the data set.

EXPLORATION 4: BOX PLOT

Data on daily temperatures in two cities are given:
Daily average temperature in a Texas city in April.

61	59	65	68	82	72	77
76	73	65	65	62	70	67
57	50	62	61	70	69	64
80	77	82	75	79	71	79
87	80					

Daily average temperature in a New York city in April.

41	57	59	40	34	33	40
51	54	64	65	45	47	63
63	54	59	57	42	45	46
70	72	48	48	43	51	59
70	89					

1. Create a stem and leaf plot for each of the two cities.

2. Find the median of each of the data sets.

3. Find the mean of each of the data sets.

4. Describe some features of the individual stem and leaf that you notice. Write down differences and similarities between the two data.

5. Did you notice the lowest and the highest temperatures for each of the cities?

6. What is the range of each of the data sets?

We close this section with the **box plot,** sometimes called the box and whisker plot, which is another way of organizing data.

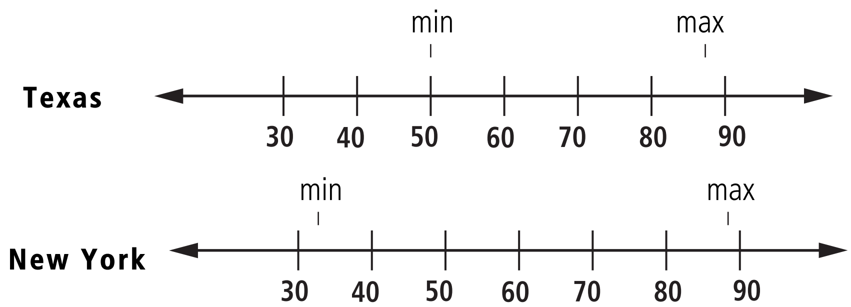
The first step is to order the data in increasing order. As an example, we use the data from Exploration 4 for the two cities.

Texas: 50, 57, 59, 61, 61, 62, 62, 64, 65, 65, 65, 67, 68, 69, 70, 70, 71, 72, 73, 75, 76, 77, 77, 79, 79, 80, 80, 82, 82, 87

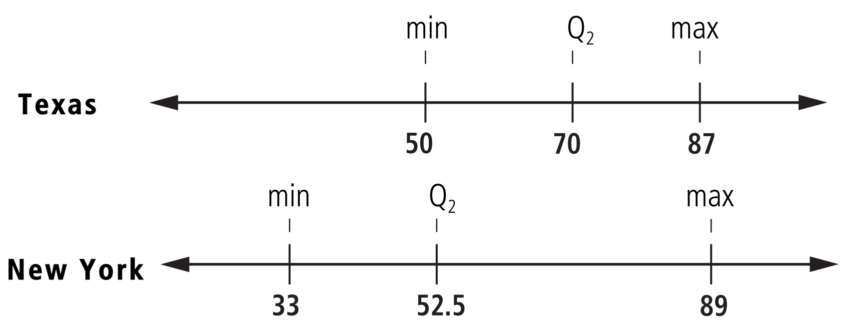
New York: 33, 34, 40, 40, 41, 42, 43, 45, 45, 46, 47, 48, 48, 51, 51, 54, 54, 57, 57, 59, 59, 59, 62, 63, 63, 64, 65, 70, 70, 89

The instructions for constructing the box plot is as follows:

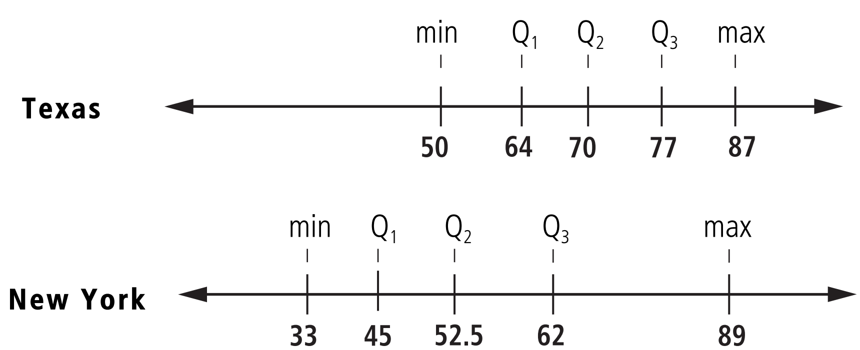
Step 1: Place the largest (max) and smallest (min) values on the respective number lines and put notches above those numbers as shown below.



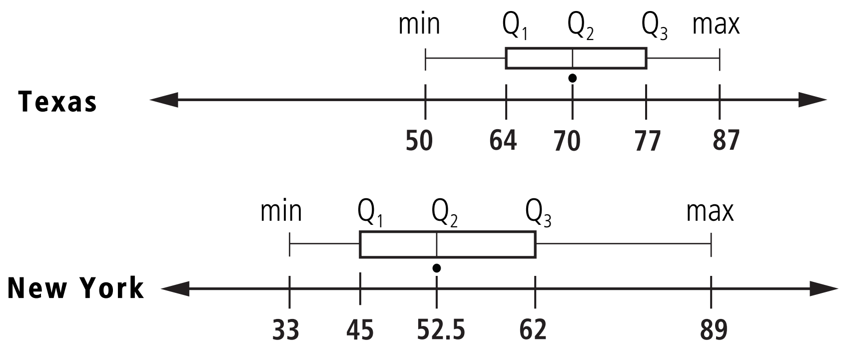
Step 2: Locate the median for the data set.



Step 3: Locate the median for the lower half called the lower or first quartile and the median for the upper half called the upper or 3rd quartile.



Step 4: Draw the graph as follows



Along with the range and medians that are summarized in the box plot, another number referred to as the Interquartile Range (IQR) tells us how the distribution of the 50% is concentrated. The IQR = upper quartile (Q_3) – lower quartile (Q_1).

EXPLORATION 5: GIANT IN THE CLASS

Using the data from Exploration 1, compute the mean and the median of the heights of Mrs. Girardeau's class.

Mean = _______ Median = _______

Now, imagine a giant who is 400 inches tall joins the class. Compute the new mean and find the new median.

Mean = _______ Median = _______

If the data is **skewed**, or uneven, a median value is a more accurate picture of the representative value than the mean is. Exploration 2 had a very tall giant join the class. The mean was affected by this **outlier**, a term used to refer to a value that is drastically different from most of the data values. The median, however, was not affected. The mean is usually more influenced by extreme values than the median.

Let us review the ways in which we summarized data in this section.

If we have a set of n values, then we can find the following measures:

- Find the mean by adding the values and dividing by n.
- Find the median by ordering the values and finding the value that is in the middle, if n is odd, or taking the average of the middle values, if n is even.
- The mode is the most frequent value that occurs. There could be two or more such values. There could also be no mode for a data set.
- The range is the difference between the largest and the smallest values in the set.
- The interquartile range (IQR) is the difference between the median of the upper half (the third or upper quartile) and the median of the lower half (the first or lower quartile).

PROBLEMS

1. Students in Mr. Davis' 6th grade math class earned the following grades on their six weeks test: {95, 30, 98, 93, 100} .

   a. Find the mean and median of this data set.

   Mean = _______ Median = _______

   b. Compare the value of each as a measure of this data.

2. In the following data set, find the mean, median, mode and range.

   {4, 9, 9, 12, 5, 9, 2, 5, 6, 7 13, 17, 5}

Mean = _______ Median = _______ Mode = _______ Range = _______

3. Create a Dot plot using the data set given in the previous problem.

4. Create a Stem and Leaf plot of the data set below. Then find the mean, median, mode and range of the data.

{28, 46, 21, 33, 46, 21, 50}

Stem	Leaf

Mean = _______ Median = _______ Mode = _______ Range = _______

5. You are given a data set represented by the following stem and leaf plot.

Stem	Leaf
10	0
9	6442
8	98777553
7	98841
6	73

Use the information to determine the following, if possible, round any value to the nearest one:

a) The mean of the data set. ____________

b) The median of the data set. ____________

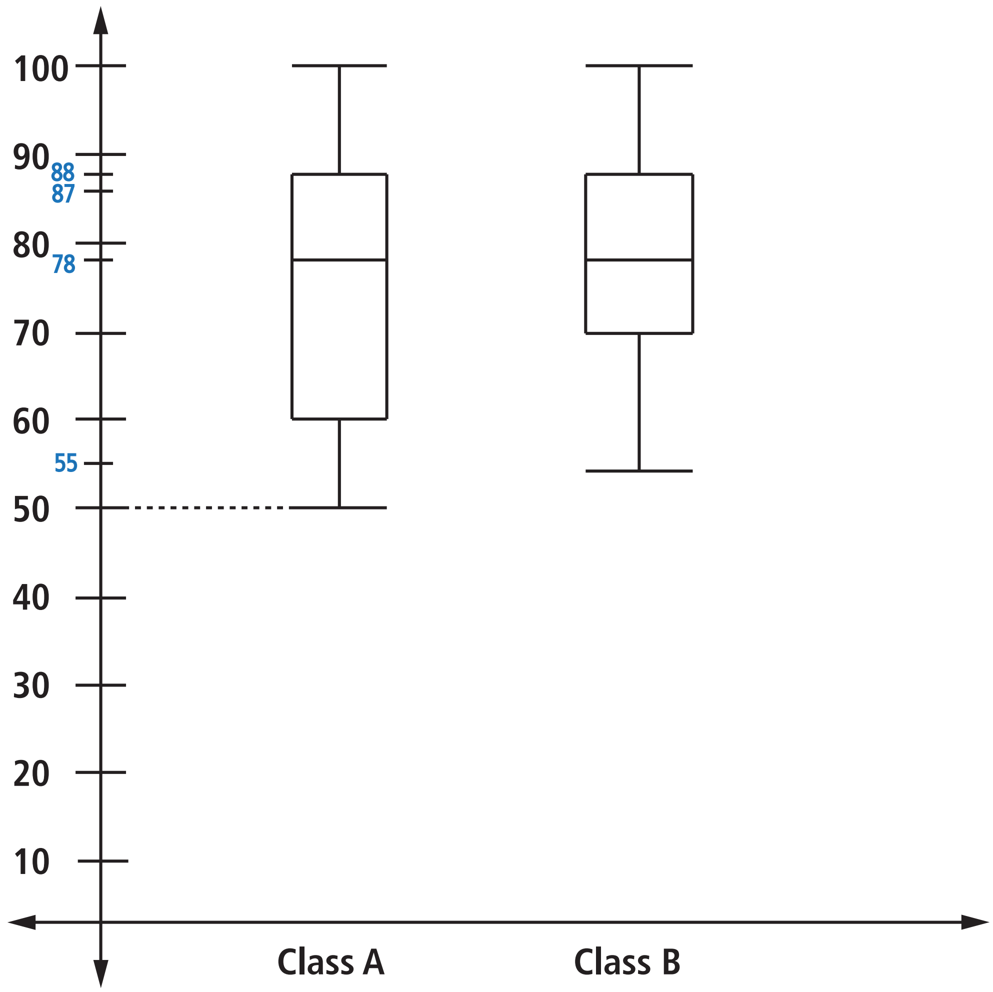
c) The mode of the data set. ____________

d) The range of the data set. ____________

e) A dot plot of the data set. ____________

f) Box plot of the data set. ____________

6. Consider the two box plots below for the test grades from two 6th grade classes. Describe the center, spread, and shape of the data distribution using the ideas of range, median, upper and lower quartiles, and the interquartile range.



SUMMARY (What I learned in this section)

DATA ANALYSIS 10

Name:_______________________________ **Date:**_______ **Period:**______

SECTION 10.2 Graphing Data

VOCABULARY

DEFINITION	EXAMPLE
Bar Graph:	
Double Bar Graph:	
Circle Graph/Pie Graph:	
Line Graph:	
Histogram:	

Big Idea: How do you construct and read bar graphs, pie graphs, and histograms?

When collecting data, it is often useful to draw a picture or graph to represent the data that has been collected. A graph of the data gives a quick, easy way to see what the data represents.

EXPLORATION 1: BAR GRAPH

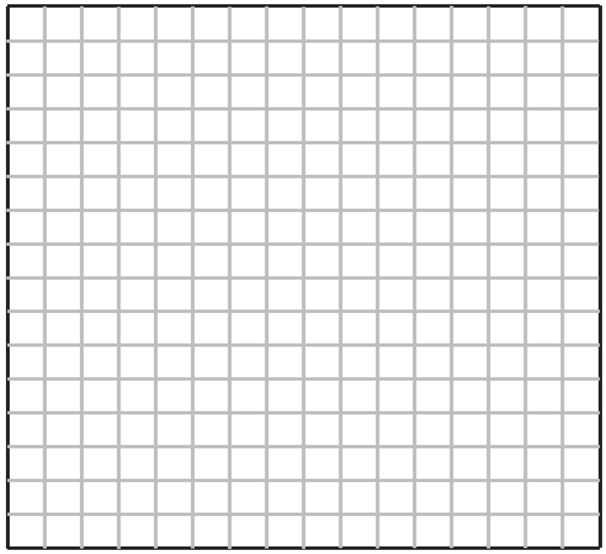
Ms. Tate's class has twenty-five students. Each student was asked which color they like best. The survey shows that 10 students prefer red, 8 students prefer green, 4 students prefer blue and 3 students prefer purple. What are the best ways to represent or display this information?

One way to display the data is to make a special kind of graph called a **bar graph**. Let's make a bar graph to represent the data from Ms. Tate's survey.

1. To construct a bar graph, draw an x- and y-axis on the grid that follows the instructions.

2. Subdivide the x-axis into four equally-spaced intervals and label the intervals with the categories Red, Green, Blue and Purple.

3. Label the y-axis with points from 0 to 12.



4. For each color, draw a vertical bar equally separated from the other bars. The height of each bar should represent the number of people who liked a particular color best.

Why does the vertical axis have a number scale from 0 to 12?

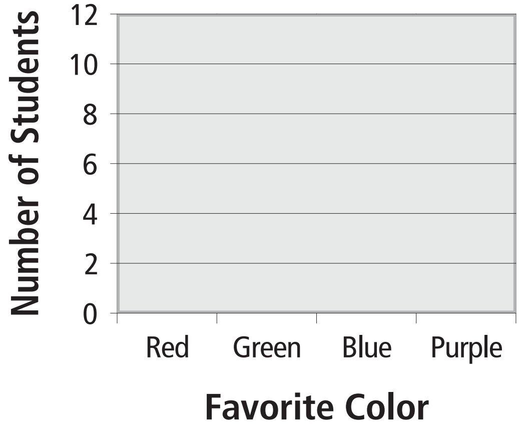
Why is there no number scale on the horizontal axis?

Is the order or the colors important?

EXPLORATION 2: DOUBLE BAR GRAPH

Sometimes we want to compare data of two different groups. Let's look again at the graph you constructed in Exploration 1. This was based on data collected from Ms. Tate's class. Suppose Mr. Riley's class was performing the same survey and want to compare their data to Ms. Tate's class. Mr. Riley's data set is as follows: 7 students prefer red, 10 students prefer green, 3 students prefer blue and 6 students prefer purple.

Following the instructions below, let's add this data to the graph by drawing a second vertical bar beside each of the four bars representing Ms. Tate's data.



1. Reconstruct the bars to represent the data of Ms. Tate's class. Choose a color and color all of the bars with that same color.
2. Adjacent to each of the vertical bars, draw a second vertical bar to represent the data from Mr. Riley's class. Choose a second color for all of these bars.
3. Create a key to show what each color represents.
4. Most importantly, title your double-bar graph appropriately.

List some examples of situations in which a double bar graph be useful.

EXPLORATION 3: CIRCLE GRAPH

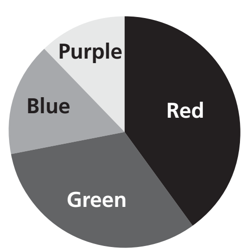
Another way to represent the data is to use a circle graph, or pie graph, to represent percentages of the data.

Because there are 25 students in Ms. Tate's class, $\frac{10}{25}$ of the class likes the color red. Convert $\frac{10}{25}$ to the decimal 0.40 and then to the percent 40%.

Calculate the percentages of the other three colors.

Red = 40%   Green = _______%   Blue = _______%   Purple = _______%

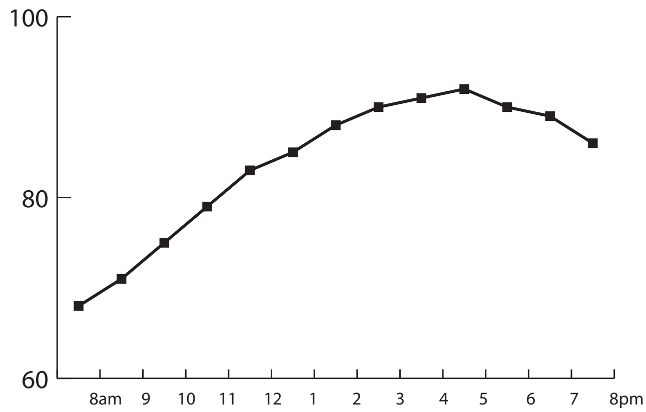
Let's examine the circle graph below.



The proportion of the circle graph with a given color corresponds to the percentage of students who prefer that color. The larger the sector of the circle graph, the greater the percentage of people who liked the color. The completed pie graph clearly represents which color students like best and makes the results of the survey visually obvious.

EXPLORATION 4: LINE GRAPH

A **line graph** for a set of data points is often used to show changes in the data over a period of time. For example, hourly changes in the temperature for one day using a line graph shows the rise and fall of the temperature. While only hourly changes are recorded, the points are usually connected from point to point as in a portion of a line graph below:



EXAMPLE 1

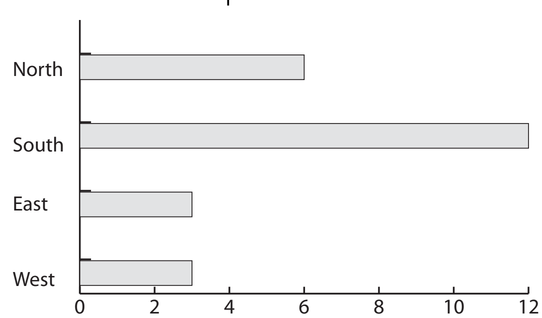
The rainfall record for a region over an 8-year period from 1990 to 1997 is listed to the right.

a. Make a bar graph to represent this data.
b. Plot the data on a coordinate plane. Label the horizontal axis to represent time in years, and the vertical axis to represent inches of rainfall. To convert the set of points to a line graph, connect the points sequentially with straight lines.
c. What differences do you notice between the line graph and the bar graph?

Year	Rainfall
1990	30 inches
1991	32 inches
1992	24 inches
1993	18 inches
1994	28 inches
1995	36 inches
1996	42 inches
1997	31 inches

PROBLEMS:

1. The bar graph represents the number of student in Mr. Mungia's class who live in various parts of town.



Use the information to determine the following:

a. How many students live in the northern part of town?

b. What percent of the students in Mr. Mungia's class live in the northern part of town?

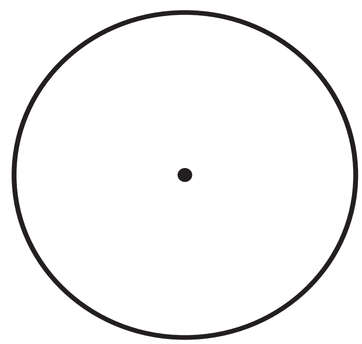
c. What percent of the students in Mr. Mungia's class do not live in the northern part of town?

d. Use the bar graph to create a corresponding pie graph next to it.

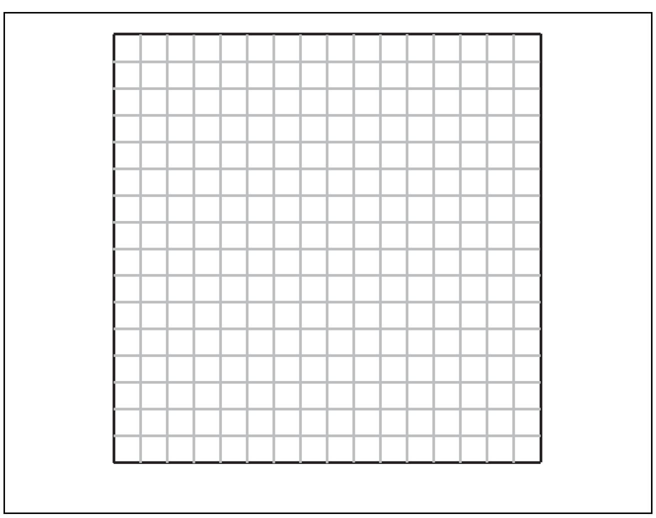
2. Mr. Kellerman made a list of the type of library books checked out by 6^{th} graders during the month of March. His data shows that these students checked out 60 mysteries, 30 science fiction books, 12 biographies and 8 books on poetry or prose.

   Create a pie chart to represent Mr. Kellerman's data. Use the chart below to organize your work. Remember to title and label your circle graph appropriately!

	# of Books	Fraction	Percent
Mystery			
Science Fiction			
Biography			
Poetry/Prose			



3. The table below represents the amount of money saved by LuAnn for the first six months of 2011. Create a line graph to accurately represent this data. Be sure to title your graph and label the axes appropriately!



Month	**Amount Saved**
Jan.	$46
Feb.	$42
March	$39
April	$64
May	$42
June	$40

a. What is the mode of dollars saved over the six month period? ________________

b. What is the range of dollars saved over the six month period? ________________

c. What is the median of dollars saved over the six month period? ______________

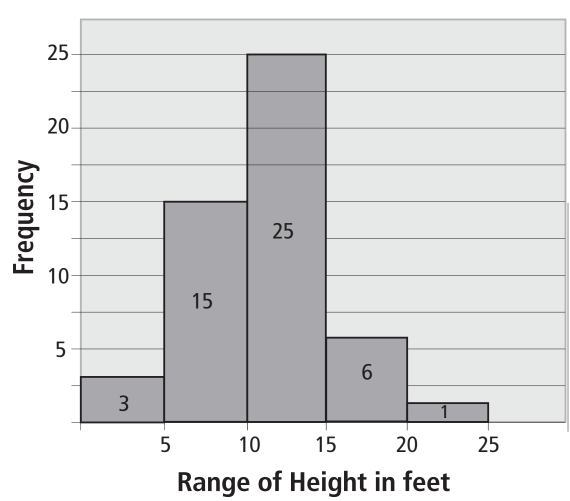
d. What is the mean of dollars saved over the six month period? ________________

A **histogram** is a graphical representation of data with numerical categories. Histograms are drawn as in a bar graph with the positive x-axis indicating numerical categories of numbers or range of numbers. The heights of the "bars" can be either the frequency or numbers in each category or they can be percent of the data. The bars are also drawn touching each other whereas the bar graph generally has the bars not touching each other. We use the following example to demonstrate these concepts.

The data shows the results of a survey taken by the city to determine the heights of the crape myrtle trees in the city.

Height of tree (in feet)	Number of trees
Between 0 and 5	3
Between 5 and 10	15
Between 10 and 15	25
Between 15 and 20	6
Between 20 and 25	1

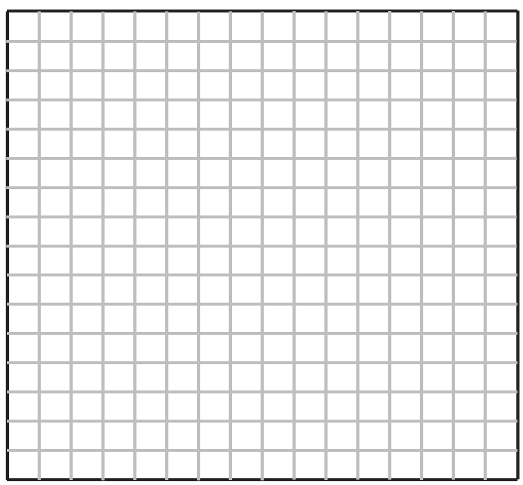
The histogram using the range of heights of the trees on the x-axis and the frequency of that size tree along the y-axis we have the following histogram:



PROBLEM

The test grades on the Test 1 and Test 2 are given below. Create a histogram for each test using the data given below.

Range of test grade	Number of Students TEST 1	Number of Students TEST 2
Between 50 and 59	2	1
Between 60 and 69	4	3
Between 70 and 79	10	2
Between 80 and 89	6	7
Between 90 and 100	3	12



Use the histograms to make observations about the shape and distribution of the data. Is there enough information to determine the mean or the median? Explain why.

SUMMARY (What I learned in this section)

DATA ANALYSIS 10

Name:________________________________ Date:_______ Period:______

SECTION 10.3 Probability

VOCABULARY

DEFINITION	EXAMPLE
Experiment:	
Outcomes:	
Sample Space:	
Simple Experiment:	
Compound Experiment:	
Tree Diagrams:	
Event:	
Simple Event:	
Compound Event:	
Probability:	

Theoretical Probability:	
Empirical Probability/Experimental Probability:	

Big Idea: How do we compute probability of events?

The study of probability allows us to make educated guesses about what might happen in the future based on past experience and to determine how likely different outcomes are. This knowledge can help us make the best choices.

In your own words, describe the meaning of the following statements. Do you think they are likely or not very likely to happen?

a. There is a 50-50 chance of getting heads on a coin flip. ______________________________

b. There is an 80% chance of rain today. ______________________________

c. There is a 25% chance of rain tomorrow. ______________________________

We will begin our study of probability with some useful vocabulary. An ___________________ is a repeatable action with a set of ____________________. For example, the experiment of flipping a coin has two possible outcomes, either head or tails. The set of all possible outcomes of an experiment is called the ______________ __________. Flipping one coin is called a __________________ _______________, because only one thing happens.

One important characteristic of an experiment is that it must be repeatable, with similar possible outcomes. In studying an experiment, the question is, "What are all the possible outcomes?" The key to finding any probability is to determine the likelihood of each possible outcome. Let's try performing some experiments now.

EXPLORATION 1: ROLLING A NUMBER CUBE

1. Take a six-sided number cube and examine the numbers on all six sides. Jot these down as possibilities of what can come up in a roll. ____

2. Roll the number cube 20 times. Use the space below to record the number that comes up for each roll. Title this experiment: **Roll One**. Keep a careful record.

Roll One

______ ______ ______ ______ ______ ______ ______ ______ ______ ______

______ ______ ______ ______ ______ ______ ______ ______ ______ ______

3. Take two six-sided number cubes of different colors such as one red number cube and one green number cube. What are the possible rolls that you could get? How many outcomes are possible?

4. Role the two number cubes 20 times. Record the numbers that come up for each roll. Be careful to record which number came up with which color number cube. Call this experiment: **Roll Two**.

Roll Two

Red	Green

Red	Green

Red	Green

Red	Green

Did you get any of the rolls that you thought you could get? Later in this section, we will refer back to the results you recorded.

EXPLORATION 2: PICK A CARD

1. Take a standard deck of cards. Examine how many cards are in the deck and note other characteristics of the cards.

   Describe all you notice, including how many cards there are with the various characteristics.

   If you picked one card from the deck, what are all the possible selections you could get?

2. Shuffle the cards carefully. Select a card and record its color and then return the card back to the deck and shuffle. Do this action 20 times and keep a record of the color of the card each time. (It is often helpful to use abbreviations to record results of an experiment.)

Pick a Card

_____ _____ _____ _____ _____ _____ _____ _____ _____ _____

_____ _____ _____ _____ _____ _____ _____ _____ _____ _____

Later in this section, we will refer back to the results you recorded.

EXPLORATION 3: COIN TOSS

1. Take a coin and observe the two sides. We call one side Heads and the other side Tails. Can you see why?
2. Flip a coin 20 times and record the results of each toss.

Coin Toss

_____ _____ _____ _____ _____ _____ _____ _____ _____ _____

_____ _____ _____ _____ _____ _____ _____ _____ _____ _____

3. As another experiment, flip the coin twice and record the first and second toss in that order. Repeat this 20 times.

Two Flip

Flip 1	Flip 2

Flip 1	Flip 2

Flip 1	Flip 2

Flip 1	Flip 2

Later in this section, we will refer back to the results you recorded.

EXPLORATION 4: SPINNER

Find or make a spinner with at least 4 different possibilities. Spin 20 times and record where the spinner lands each time. In the space below, create your own table and record your outcomes.

Later in this section, we will refer back to the results you recorded.

EXPLORATION 5: TREE DIAGRAMS

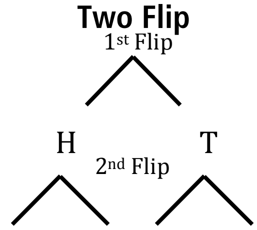
It is often helpful to draw tree diagrams to represent the outcomes of an experiment. For example, when flipping a coin, we often write H for the outcome of getting a head and T for the outcome of getting a tail. The sample space is the set {H, T}; there are only two possible outcomes.

EXAMPLE 1

In the "Two Flip" experiment, you flipped a coin once and observed the outcome, then flipped it a second time and observed that outcome. You then recorded the outcome. What is the set of all possible outcomes? How many outcomes show no tails? Let's create a tree diagram to find the answers to these questions.

The first flip outcomes have been done for you. Complete the tree diagram to show all possible outcomes.

Two Flip



The order of outcomes is important. The outcome of getting a head and then a tail, denoted by HT, is a different outcome from getting a tail and then a head, denoted by TH.

a. How many possible outcomes does this experiment have? ______________________

b. List the sample space of this experiment.: ____

c. Would you describe this as a simple experiment or a compound experiment? Why?

d. How many outcomes show no tails?

____

In both the Coin Toss and Two Flip experiments, each outcome in the sample spaces has the same chance of occurring as any other outcome. Each outcome is then said to be **equally likely**.

EXPLORATION 6: SIMPLE VERSUS COMPOUND EVENTS

DEFINITION 10.2: Probability
In an experiment in which each outcome is equally likely, the **probability of an event A**, written **P(A)**, is $\frac{m}{n}$, where m is the number of favorable outcomes and n is the total number of outcomes in the sample space S.

Notice that the probability of an event from an experiment is always a number between 0 and 1. A probability of ______ indicates that an event cannot occur and a probability of ______ indicates that an event will certainly occur.

Explain why $P(S) = 1$, where S is the sample space for an experiment.

The words simple and compound are used to describe both events and experiments. The main thing to remember is that a **simple event** has just one outcome in a set while a **compound event** has more than one outcome listed. A **simple experiment** has just one action, such as pick a card, roll a number cube, or flip a coin. A **compound experiment** involves more than one action such as roll two number cubes or flip a coin more than once.

EXAMPLE 2

Consider the Two Flip experiment. Let's define E as the event of your coin landing on heads at least once.

a. What possible outcomes satisfy the criteria for being in E? In other words, E = {____________________________}

b. Because E contains more than one possible outcome, it is a __________________ event.

Sometimes when we study a compound event like E, it is useful to find the possible outcomes that are not in E. We call this set the **complement of E**, or E^C. For example, in the Two Flip experiment, we have the sample space S = {HH, HT, TH, TT}. We called an event E in this experiment as "getting at least one head." This can be written as E = {HH, HT, TH}. The complement of E consists of all the outcomes in the sample space when you do not get at least one head. This leaves only TT where there are no heads. We then write E^c = {TT}.

In an experiment in which each outcome is equally likely, the probability of an event A, written $P(A)$, is $\frac{m}{n}$, where m is the number of favorable outcomes and n is the total number of outcomes in the sample space S.

In Example 1, the probability of showing no tails is $\frac{1}{4}$, because 1 of the 4 equally likely outcomes shows no tails. The probability of showing at least one tail is $\frac{3}{4}$. Notice that $P(S) = 1$ and that $P(E^C) = 1 - P(E)$. See if you can explain why.

EXAMPLE 3

Consider the Roll One experiment from Exploration 1.

a. What is the probability of getting a three?

Let A = the event of rolling a three in this experiment.

$P(A) =$ ____________

b. What is the probability of not getting a three?

Let B = the event of not rolling a three in this experiment.

$P(B) =$ ____________

Notice that B is the complement of A, because together they make up the entire sample set of the experiment.

c. What is the probability of getting an even number?

Let $C =$ ____

$P(C) =$ ____________

d. What is the probability of getting a number greater than 4?

Let $D =$ ____

$P(D) =$ ____________

EXPLORATION 8: THEORETICAL PROBABILITY

When we "consider" an experiment of rolling one number cube, we do not actually roll a number cube. Instead, we think about what could possibly happen if we rolled a number cube. This is called a thought experiment and is used in **theoretical** probability. If we really rolled a number cube and used the observed outcomes as in the first activity, that would be an example of **empirical (or experimental)** probability.

EXAMPLE 3

Let's look at an example of theoretical probability. List the possible outcomes of rolling a red number cube and a green number cube. Organize your data in the table below.

		Green Number Cube					
		1	2	3	4	5	6
Red Number Cube	1	{(1,1)					
	2						
	3						
	4						
	5						
	6						(6,6)}

a. How many possible outcomes are there?

____

b. Your sample space includes (3, 4) and (4, 3). Explain why rolling a 3 and a 4 is different from rolling a 4 and a 3.

c. You may notice that this example is similar to the Roll Two experiment in Exploration 1. How is this example different?

PROBLEMS

1. Consider again the Roll Two experiment. What is the probability of each of the following events?

   a. A = {getting at least one 6}

   $P(A)$ = ______________

   b. B = {getting a "double"}

   $P(B)$ = ______________

   c. C = {the sum of the two number cubes is 7}

   $P(C)$ = ______________

2. Ishan draws one card from a standard deck of 52 playing cards.

   a. What is the probability of drawing a Queen? ______________________________

   b. What is the probability of drawing a club or spade? ____________________

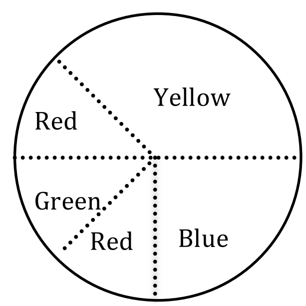
   c. What is the probability of drawing a diamond? __________________________

3. Use the spinner below to answer the following questions.

   a. What is the probability of spinning red or blue?

   b. What is the probability of spinning a yellow?

   c. Describe the complement of the event listed in item a.



____

____

   d. Describe the complement of the event listed in item b.

____

____

SUMMARY (What I learned in this section)

Name:________________________________ **Date:**_______ **Period:**______

SECTION 10.4 Rule of Product and Rule of Sum

VOCABULARY

DEFINITION	EXAMPLE
Independent Events:	
Rule of Product:	
Rule of Sum:	

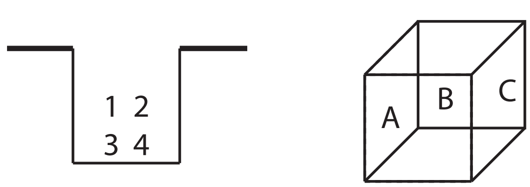
Big Idea: What are the Rule of Product and Rule of Sum?

EXPLORATION 1: USING A TABLE TO SHOW THE SAMPLE SPACE

One of the major goals of mathematics is to find simple underlying ideas to explain how and why things work. To do this, mathematicians analyze problems by breaking them into smaller steps.

Suppose you have a hat and a box.

The hat contains identical cards each with a number 1, 2, 3, or 4 written on them, and the box contains identical cards each with a letter A, B, or C written on them, as illustrated below:



Imagine the following experiment:

Without looking, reach into the hat and pull out one number card, and then reach into the box and pull out one letter card. Now look at the two cards.

What is a possible outcome?

____

List all the possible outcomes. ____

____

____

How many possible outcomes are there?

____

To answer the last question, let's create a table to list all the possible outcomes and count to see the number of possible outcomes.

	A	B	C
1			
2			
3			
4			

What is the size of the sample space?

____

How does the arrangement in the table help to count the number of outcomes? ______________________

____

____

____

____

EXAMPLE 1

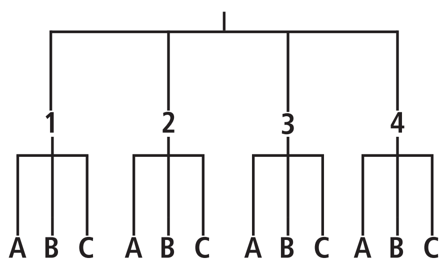
Suppose you roll a number cube and then draw a card, without looking, from a set with one red, one white, and one blue card. Use the space below to create a table of all possible outcomes.

What is the size of the sample space?

EXPLORATION 2: USING A TREE DIAGRAM TO SHOW THE SAMPLE SPACE

A tree diagram is another method to visually represent the sample space.

Recall the hat and box experiment in Exploration 1: A hat contains the numbers 1, 2, 3, and 4. A box contains the letters A, B, and C. Without looking, you were to draw a number from the hat, and then a letter from the box. Displaying this information in a tree diagram would look like this:



The first action in the experiment is illustrated by the numbers 1, 2, 3, and 4 listed as the first set of branches on the tree diagram. After choosing a number, however, you are to draw a letter from the box. This action is shown by the three branches below each number. For example, if you drew the number one, you could then draw an A, B, or C.

How many outcomes are possible? ______________ How do you know?

___

___

Now it's your turn!

EXAMPLE 2

You are going camping with your friends and decide to pack lightly since you have to carry your bag as you hike to the campground. You pack a pair of khaki cargo shorts, a pair of jeans, and a pair of navy shorts. You take a yellow T-shirt, a red T-shirt, and a long-sleeved denim shirt. What are all the possible outcomes for selecting a bottom and a top to wear? Create a tree diagram to illustrate your sample space.

You can create ________ different outfits to wear.

EXPLORATION 3: THE RULE OF PRODUCT

When the first action in a compound experiment has no effect on the second action (or subsequent actions), we say the events are ________________________________. In such events, the actions can occur either in succession or simultaneously, it does not matter which.

Looking at the table you created in Exploration 1 we see 4 rows of possible numbers and 3 columns of possible letters that you could select. Mathematically, we could express this a 3+3+3+3 *or* ____________, which equals _____.

Look now at the tree diagram listed in Exploration 2. You see there are 4 numbers with 3 branches off each. As before, $3+3+3+3 = 3 \cdot 4 =$ ______.

This process is a formal rule in counting:

Theorem 10.1: The Rule of Product
If one action can be performed in m ways and a second independent action can be performed in n ways, then there are $m \cdot n$ possible ways to perform both actions.

What does this rule mean? Explain.

____

____

____

EXPLORATION 4: THE RULE OF SUM

Return again to the hat of number cards and box of letter cards. We're going to alter the experiment just a little. Suppose that, you take all of the number and letter cards and put them into a bag. Then without looking you pull out one card. How many possible outcomes are there for this experiment?

The main difference between the earlier situation and this one lies in the change of one word. In the first example we chose a number card *and* a letter card, while in the second we chose either a number card *or* a letter card. This demonstrates the importance in mathematics of carefully reading words, especially words like "and" and "or". The following rule captures the number of ways to perform one action *or* another:

Theorem 10.2: The Rule of Sum
If one action can be performed in m ways and a second action can be performed in n ways, then there are $(m + n)$ ways to perform one action or the other, but not both.

What does this rule mean? Explain.

____

____

____

PROBLEMS

1. Marty was going to buy a ball and a magazine. He had to choose between a soccer ball, a basketball, or a football. He could choose a game magazine, sports magazine, or comics magazine.
   a. Construct a tree diagram to show the sample space of all the possible outcomes.

b. List all the possible outcomes

____

____

c. Use the rule of products to obtain the possible outcomes.

____

d. If Marty were to buy either a ball or a magazine, what are all of the possible outcomes?

____

2. George will win if he draws either a red even number or a red face card from a standard deck of cards. Find the following favorable outcomes:

a. even red cards ______________
b. red face cards ______________
c. possible outcomes that will allow him to win (use rule of sum). ______________

3. Recall the experiment in Exploration 1: A hat contains the numbers 1,2,3, and 4. A box contains the letters A, B, and C. The experiment is to pick a number from the hat and then pick a letter from the box. Use the sample space to compute the following probabilities:

a. P(drawing an even number and an A). ______________
b. P(drawing neither a 1 nor an A). ______________
c. P(drawing a 1 or an A). ______________
d. P(drawing an odd number or a B). ______________

4. You are making a sandwich and have the following to choose from: white or wheat bread, ham, turkey, or tuna, American or Swiss cheese. What are all the possible outcomes if you choose one type of bread, one type of meat, and one type of cheese?

How many possible outcomes are in the sample space? ______________

5. You have a number cube with the digits 1-6, and a spinner with three equal sections of red, white, and blue. Create a tree diagram or a table to show all the possible outcomes.

   a. How many outcomes are possible if you roll the number cube and then spin the spinner?______________________________

   b. What is the probability of rolling a 2 and spinning blue? _________________________

   c. How many outcomes are possible if you roll the number cube or spin the spinner?

   ____

6. You must create a password using three different letters and one numerical digit. You decide that the letters will come from the name of your favorite pet finch, ROY, listed in any order, and you will select one of the digits from your favorite number, 76. Make a table or a diagram to show all the possible outcomes.

   There are __________________ passwords possible.

7. You are challenged to find the probability of certain events happening in this experiment: You roll a standard 6-sided number cube, then spin a spinner with three equal sections colored red, white, and green, and then pull a letter A, B, or C from a hat. First make a table or a diagram to show all the possible outcomes.

   a. How many outcomes are possible? _______________

   b. What is the probability of rolling a 1, spinning red, and drawing an A? _______________

   c. What is the probability of rolling a prime number, spinning white, and drawing a letter that is not A? _______________________

   d. What is the probability of rolling an even number, spinning green, and drawing a C? _____________________________

SUMMARY (What I learned in this section)

Name:______________________________ Date:________ Period:_______

CHAPTER 10: SPIRAL REVIEW

1. Find the number of possible lunch plates you could make if you picked one item from each column:

Hamburger	Potato chips	Soda
Hot Dog	French Fries	Iced Tea
Grilled Cheese	Fruit Cup	Water

You could make ___________ different lunch plates.

2. Find the mean, median, mode, and range of the following set of numbers: {12.4, 9.411, 12.09, 12, 11.099}

Mean:_____________ Median: _____________ Mode: _____________ Range: _____________

3. Evaluate the following expressions using the Order of Operations:

$(39 - 21) \div 6 \cdot 5 + 3^3$

____

____

____

____

$3 \cdot 9 - 4^2 + 5.75$

____

____

____

____

4. Alexa's grandmother is canning jelly. She buys jars that are sold 12 to a box and lids that come 8 to a box. She wants to buy the least number of lids and jars so that she will have exactly one jar per lid. How many boxes of jars and lids will she need?

Alexa's grandmother will need _______ boxes of jars and ________ boxes of lids.

5. Evaluate each expression for the given values of n.

n	**36 – *n***
1.1	
2.2	
3.3	
4.4	

n	**$7 + n^2$**
1	
2	
3	
10	

6. Theresa needed $\frac{7}{10}$ of a yard of canvas to make a cover for her chair. She bought a piece that was $\frac{5}{6}$ of a yard. Will she have enough canvas to make the chair cover? ___________

   Will any canvas be left over? ____________ If so, how much? _________

   If not, how much more will she need? _____________

7. What is the volume of a cube that has a side measure of 2.5 feet?

   The volume of the cube is ________________________________.

   (Indicate the units)

8. Caelen has $14\frac{7}{8}$ yards of rope that he wants to cut into $\frac{1}{4}$ yard pieces for an art project. How many pieces will he be able to cut from the rope?

   Caelen will be able to cut _________ pieces of rope.

9. Connor practices Spanish 3 hours every 2 weeks. Talon practices the trombone 5 hours every 4 weeks. If Connor and Talon continue to practice Spanish and the trombone at this rate, what is their combined number of hours that they spend on the two activities in one year?

Connor and Talon will spend __________ hours practicing in one year.

10. Mr. Wozniak has two flowerbeds in which he wants to plant roses and tulips. He wants the ratio of tulips to roses to be the same in each bed. He planted 10 tulips and 6 roses in the first flowerbed. How many tulips will he need for the second bed if he planted 15 roses?

Mr. Wozniak will need ________ tulips.

MATH OF FINANCE 11

Name:______________________________ Date:________ Period:______

SECTION 11.1 TYPES OF CHARGE CARDS

VOCABULARY

DEFINITION	EXAMPLE
Credit card:	
Debit card:	
Interest rate:	
Late fee:	
Monthly fee:	
Deposit:	
Withdrawal:	
Transfer:	

Big Idea: What are the advantages and disadvantages associated with different types of charge cards and how do you balance a checking account?

One internet site with helpful information is at:

http://www.nytimes.com/2009/01/06/your-money/credit-and-debit-cards/primercards.html

DEBIT CARDS

EXPLORATION 1

Fill in the table to compare advantages and costs associated with debit cards offered by two different local financial institutions. Which features are most important to you? Which card seems to offer the best value?

Ask your parents which type of card they use the most. Why do they prefer that type of card? Are there different fees associated with each? What are these fees? Do their cards also have special bonuses to encourage them to use any of their cards?

	ADVANTAGES	COSTS
Institution 1		
Institution 2		

EXAMPLE 1

Sue opened an account on January 15 by depositing $250 into a new savings account. On January 18, she made a withdrawal of $100, and on January 24, she transferred $75 from her savings account to her checking account to pay the balance she owed in that account. What is her new savings account balance? Show your work.

One internet site with helpful information is at:

http://banking.about.com/od/checkingaccounts/ss/balancechecking.htm

CHECK REGISTER

Suppose Andy has a checking account with a $800 balance. He has the following income and expenses. Create a check register to record the transactions and keep a running balance of what Andy has at the end of each transaction.

January 28	Mowing the lawn for the month	$25
February 12	Valentine candy check #242	$15.25
March 5	Birthday money from parents	$40
April 2	Monthly allowance	$20
May 15	Holiday gift check #243	$24.75

Check #	Date	Transaction Description	$ Withdrawal	$ Deposit	$ Balance
					$800.00

SUMMARY (What I learned today)

MATH OF FINANCE 11

Name:________________________________ **Date:**_________ **Period:**______

SECTION 11.2 CREDIT REPORTS

VOCABULARY

DEFINITION	EXAMPLE
Credit history:	
Credit report:	
Credit score:	

Big Idea: How do we create a positive credit history and how do we interpret a credit report?

Several Internet sites with helpful information are:

http://www.ftc.gov/bcp/edu/microsites/freereports/index.shtml

http://www.ftc.gov/bcp/edu/pubs/consumer/credit/cre03.shtm

EXPLORATON 1

A. What is a person's credit history?

B. What is a person's credit report?

C. How does a consumer get a credit score?

D. Investigate and then answer the following questions about credit scores:

1) What is the range of the credit scores?____________________

2) What is considered a positive score? ____________________

3) What is considered a negative score? ____________________

EXPLORATON 2

A. Name three credit-reporting agencies.

1)

2)

3)

B. What is the time limit for keeping negative financial information?

C. Why do you need a positive credit history?

SUMMARY (What I learned today)

MATH OF FINANCE 11

Name:________________________________ **Date:**_________ **Period:**_______

SECTION 11.3 GOING TO COLLEGE

Big Idea: How do we create a positive credit history and how do we interpret a credit report?

One internet site with helpful information is at:

http://nces.ed.gov/fastfacts/display.asp?id=76

What is the difference between a Junior College versus a 4-year College? Name any advantages or disadvantages of attending one or the other?

EXPLORATION

I. Access information about the costs of attending college.

1) Locate 2 junior colleges and their tuition for one year.

Junior College Name	Tuition for one year

2) Locate 2 private 4-year colleges and 2 state 4-year colleges and their tuition for one year.

Private 4-year College Name	Tuition for one year

State 4-year College Name	Tuition for one year

II. Access information from the Internet or financial institutions to summarize the following 5 options to finance your college education.

1) **Savings**

2) **Grants**

3) **Scholarships**

4) **Student Loans**

5) **Work Study**

Can you think of any other ways to help finance your college education?

SUMMARY (What I learned today)